Imperfect IPR Enforcement, Inequality, and Growth

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Abstract

This paper addresses the concern of how an imperfect patent protection regime affects the long-term economic performance. To this aim a ladder quality R&D based growth model is adopted, in which the institutional setting for patent protection directly impacts the long-run incentive to innovate as well as to invest in human capital. By ruling, Courts' interpretation of patent law can generate the coexistence of both top quality leader's and follower's product. The existence of such an imperfect IPR enforcement against followers constitutes a threat for the quality leader monopoly position, affecting its strategic behavior. The results show that a tighter IPR enforcement - also in a subset of the existing industry lines - increases both skill premium and inequality in consumption level between an unskilled and skilled population. Moreover, a tighter IPR enforcement increases the innovation rate and the per capita output growth rate. More interestingly, the existence of an imperfect IPR enforcement allows an IPR "enforcement externality" between technological fields to emerge. In particular, what is more valuable for a R&D firm - in term of a higher innovation rate - is a relatively tighter IPR enforcement in other industry lines rather than in its own productive line.

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1 Introduction

The importance of both R&D and human capital for technological progress and for long-run economic performance is widely recognized by both academic and non academic analyses. A diffuse legal instrument used to spur R&D investments consist in granting a patent protection to valuable innovations. Indeed, intellectual property rights (IPR), and patent law in particular, play a primary

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role for long-run innovation and growth performance.[1] There was a 71% increase in patent grants from 1978 to 1995. Yet, in practice a patent law does not allow the patent-holder to obtain a perfect protection against potential rival firms. As remarked by Shapiro (2003), a patent is best viewed as a probabilistic property right. Indeed, the author writes that nothing in the patent grant guarantees that the "defendant in the patent suit will be found to have infringed." (Shapiro, 2003, p. 395). In the U.S., for instance, it is both the Federal Circuit Judges and the Supreme Court who decide how to interpret and apply the patent law. As remarked by Scotchmer (2004), litigation gives courts an opportunity to modify the design of patent law itself. Indeed, the Court of Appeals in the Federal Circuit was instituted in the U.S. in 1982 partly because of dissatisfaction of patent litigants about a lack of expertise and consistency on the part of federal judges.

In particular, a patent application must indicate the claims of the patent that may cover a product, apparatus, process, composition of matter, or design, so that when a product or technology falls within at least one of the patent's claims and thereby infringes, a decision must be made as to whether to fight or not. This decision is also based upon the plaintiff's probability of success and in avoiding infringement future activities. According to Lanjouw and Schankerman (2001), patent litigation has grown dramatically during the period 1978-1999, and this is attributable both to the overall increase in patenting and to the changing composition of patents. The authors find that the overall rate of litigation is about 19 filed suits per 1,000 filed patents. Yet, the rate of litigation vary substantially across industries and technological fields. The lowest rate are in chemicals (about 12), pharmaceuticals are only a little higher than the average, but computers and biotechnology are much higher. Most of the increase in the patent suits has been in drugs, biotechnology and computers and other electronics, which traditionally have always been highly litigated.[2]

Very interestingly, the low rate of filed suits - even if highly heterogeneous between technological fields - does not mean that patent infringements are correspondingly low, and that patent enforcement mechanisms are so effective. Indeed, Lanjouw and Schankerman (2001) find that ninety-five percent of filed patent suits are settled before trial. There is a strong incentive to settle even if parties do not agree on the prospects, and the dispute is legitimate because litigation is very costly.[3] On such concerns, they state that: "From a policy per-

[1] See, among others, Romer (1990), Anant, Dinopoulos and Segerstrom (1990), Grossman and Helpman (1991), Aghion and Howitt (1992, 1998), Dinopoulos and Segerstrom (1999), Scotchmer (2004). The increased propensity of companies to file for patents has been well documented by several contributions (see Shapiro, 2003).

[2] Lanjouw and Schankerman (2001) consider the litigation rate and composition for the period 1980-1984 in the U.S. From a historical perspective, Khan (2002) shows that the setup of an efficient IPR regime had been assessed in a broader institutional context including trade and antitrust policies, with a different level of protection which took into account intersectoral differences as a part of a more general industrial policy. Indeed, while the U.S. patent system undoubtedly contributed to economic growth, its effects varied widely between different industries especially from the mid 19th century onwards.

[3] A patent litigation can cost each litigant between $1 million and $3million per suit, or about $500.000 per patent claim (see Scotchmer, 2004). Moreover, Lerner (1995) and Lanjouw

spective, this is good news because it means that enforcement of patent rights relies on the effective threat of court actions (suits) more than on extensive post-suit, legal proceedings that consume court resources". Moreover, the same authors show that, in each technological group, infringement suits - i.e. those suits where the patent owner is the plaintiff - account for the bulk of litigation, about 84% if unclassified cases are excluded, and 90% if they are treated as infringement suits.[4] In addition, Lanjouw and Schankerman (2001) find that the plaintiffs' probability of winning the patent suit does not depend on the characteristics of patents and their owners among patent disputes.

According with such empirical analyses, this paper allows for an imperfect IPR enforcement, and then its effects on the long run economic performance of an economy are analyzed. To this aim a Schumpeterian ladder quality growth model with human capital accumulation à la Dinopoulos and Segerstrom (1999) is adopted. Because of the empirical evidence about a high share of patent litigation for infringement, this paper departs from standard Schumpeterian literature by admitting that patent law can not definitively determine the effective protection of a patented product against imitators. This implies that the standard complete lagging breadth assumption is relaxed by considering the possibility that Courts allow firms to enter the market with a partial imitation of the top quality product.[5] Any imitating firm endogenously decide the degree of imitation of the top quality product along any product line, and it has a lower probability to win a patent suit against the top quality patent-holder the more close its imitation is to the top quality product.

The results show that a tighter IPR enforcement spurs human capital accumulation and increases the skill premium. Moreover, both the aggregate rate of innovation and the per capita output growth rate are higher with a tighter IPR protection. Yet, because of a higher skill premium and a lower production, the consumption level inequality between an unskilled and skilled population is higher, and the unskilled population suffers of a lower consumption level than with a softer IPR enforcement.

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and Lerner (2002) provide empirical evidence that even if parties can settle their patent disputes without resorting to suits, the effective threat of litigation influences the incentive to undertake R&D by preventing small firms entering those R&D areas where the threat of litigation from larger firms is high.

[4] In these infringement suits, a concept which is often used as the most basic test for infringement, is the issue of the "use of a technology". If a product uses a technology covered by the claims of a patent, then the product infringes the patent. Yet, as remarked by O'Donoghue (1998, p. 657-658): "the use of a technology is a vague concept, so the courts has a lot of discretion in how to interpret it...the strength of lagging breadth is determined by the interpretation of the doctrines of disclosure and enablement."

[5] By partial imitation I mean a product of a lower quality than the existing state-of-the-art product, but with a quality level higher than the previous incumbent producer. Note that this definition of partial imitation coincides with what O'Donoghue (1998), and O'Donoghue and Zweimuller (2004) call inferior product. Standard Schumpeterian growth theory assumes a perfectly enforceable patent law that allows the patent-holder to enjoy a monopolistic power along a product line (see Anant, Dinopoulos and Segerstrom, 1990; Aghion and Howitt, 1992; Grossman and Helpman, 1991). Moreover, in standard Schumpeterian literature a complete lagging breadth is assumed, which means that any patentable innovation receives sufficient lagging breadth to protect the entire quality increase facilitated by the innovation.

Furthermore, according to the empirical evidence this paper allows a heterogenous IPR enforcement between the existing industry lines to be considered. A comparative static analysis shows that - whenever only a subset of the existing industry lines has a tighter IPR enforcement - the innovation rate will be higher along that industries with an unchanged IPR protection than in that lines with a tighter IPR enforcement. Therefore, in any industry line, what matters to spur the rate of innovation is a relatively tighter IPR enforcement in other industries than in own productive line. This IPR "enforcement externality" between industry lines is new in literature, and allows the empirical evidence about heterogenous IPR enforcement between industries and their innovation rate to be explained and tied.

Indeed, as indicated above, the rate of litigation vary substantially across industries and technological fields: the lowest rate are in chemicals and pharmaceuticals, and computers and biotechnology are much higher. In particular, most of the increase in the patent suits has been in drugs, biotechnology and computers and other electronics. Therefore, within this framework the existence of a lower number of patent suits for infringement in traditional fields (chemicals) than in other ones (drugs and computers and communications) can be interpreted as saying that a lower probability to be infringed, and a lower degree of tolerance by courts against imitations, exists in chemicals and pharmaceuticals than in drugs and computers.

The results of the model predict a higher innovation rate in industries that have a relatively lower degree of IPR enforcement. This prediction well fits the data presented by Hall, Jaffe and Trajtenberg (2001). They show that traditional fields (chemicals, mechanical, and others) have experienced a steady decline in their number of patents as share of total patents over the past 3 decades, while the shares of computers and communications, drugs and medical rose sharply within the same period. Moreover, the authors find that "The absolute number of patents in the traditional fields (Chemical, Mechanical and Others) declined slightly up to 1983 (certainly during the late seventies), and then increased by 20-30%. By contrast, the emerging fields of Computers and Communications and Drugs and Medical increased throughout the whole period, with a marked acceleration after 1983. All told, the absolute number of patents in C&C experienced a 5-fold increase since 1983, and similarly for those in D&M."

The paper is organized as follows. Section 2 ties the paper with the existing literature. Section 3 sets up the model. Section 4 derives the balanced growth properties of the economy. Section 5 analyzes the effect of a tighter IPR enforcement. Section 6 concludes.

2 Related Literature

This paper is a first attempt to study the long-run growth effect of an imperfect IPR enforcement against endogenous imitations of top quality product within a general equilibrium framework. Cozzi (2001) introduces an imperfect IPR enforcement along a patent race by allowing a misappropriation of an original idea in a quality ladder one sector model with homogeneous labor force. In Cozzi (2001) misappropriation happens before a patent is granted. O'Donoghue and Zweimuller (2004) adopt a general equilibrium model with vertical and sequential innovations, and they focus on the long run growth effect of other two patent instruments: patentability requirement and leading breadth. Differently from O'Donoghue and Zweimuller's (2004) contribution, this paper considers the effect of an imperfect IPR enforcement against imitators, i.e. a lagging breadth policy lever is concerned. Moreover, the effects of an imperfect patent enforcement on human capital accumulation, inequality in income and consumption level between an unskilled and skilled population are here analyzed. This paper also allows a heterogenous IPR enforcement among the existing industry lines - as the empirical evidence shows to be in practice - to be considered. O'Donoghue, Scotchmer, and Thisse (1998) considers the role of lagging breadth on the innovation incentive, and on R&D profitability within a partial equilibrium framework. Shapiro (2003) also adopts a partial equilibrium framework and he analyzes how an antitrust law can be used to evaluate different types of patent settlements with the final aim to do not harm final consumers.

3 The Model

3.1 Households

Let us households differ in their uniformly distributed personal ability $\theta \in [0,1]$ of their individual members to become skilled workers.[6] All individuals have identical intertemporally additively separable preferences for a continuum set of goods and services indexed by $\omega \in [0,1]$ produced by the private sector, and are endowed with a unit labor/study time endowment whose supply generates no disutility. The intertemporal and instantaneous preferences are described as follows

$$\int_0^\infty N_0 e^{-(\rho-n)s} \log u_\theta (s)\, ds \tag{1}$$

where

$$\log u_\theta (s) \equiv \int_0^1 \log \left[\sum_{j=0}^{j^{\max}(\omega,s)} \lambda^j q_\theta (j,\omega,s) \right] d\omega$$

represents the instantaneous utility function. The consumption value of goods and services for an individual with ability θ is defined as

$$c_\theta (s) \equiv \int_0^1 \left[\sum_{j=0}^{j^{\max}(\omega,s)} p(j,\omega,s)\, q_\theta (j,\omega,s) \right] d\omega,$$

[6] As Dinopoulos and Segerstrom (1999) all members of households θ have the same ability level equal to θ, and all households have the same number of members at each point in time.

and the intertemporal budget constraint for each individual with ability θ is

$$W_\theta(t) + Z_\theta(t) = \int_t^\infty N_0 e^{-\int_t^s [r(\tau)-n]d\tau} c_\theta(s)\, ds$$

where N_0 is the initial population size of the economy and n is its constant growth rate. ρ is the constant and common rate of subjective time preferences - with $\rho > n$ - and $r(s)$ is the market interest rate. $q_\theta(j, \omega, s)$ is ability $\theta \in [0,1]$ household's per member quantity flow of quality $j \in \{0, 1, 2, ...\}$ of good/service $\omega \in [0,1]$ at time $s \geq 0$ - $p(j, \omega, s)$ being the price of good ω of quality j at time s - $c_\theta(s)$ is the nominal expenditure. $W_\theta(t)$ and $Z_\theta(t)$ are human and non-human wealth levels. A new vintage of good/service delivers $\lambda > 1$ more quality services than its previous version. The quality jump λ is assumed to be constant over time, and it is not high enough to generate drastic innovations. Different versions of the same good ω are regarded by consumers as perfect substitutes after adjusting for their quality ratios, and $j^{\max}(\omega, s)$ denotes the time s top quality of good ω. As usually assumed in Schumpeterian literature, each consumer only buys the state-of-the-art product along any product line. In this model, production is conducted by monopolistic firms which manufacture goods and services subject to vertical technological progress, which renders new products of a better quality than previous ones. Instantaneous Bertrand competition at all dates between the incumbent and the innovating firm - jointly with the existence of non-drastic innovations - imply a limit pricing strategy by the innovating firm that allows it to be the only producer along its own product line.

Individuals are finitely lived members of infinitely lived households, being continuously born at the constant rate β, and dying at the constant rate δ, with $\beta - \delta = n > 0$. $D > 0$ denotes the exogenous given duration of their life.[7] Each individual chooses to train and becomes skilled at the beginning of her life; the duration of her training period - in which the individual cannot work - is exogenously fixed at $T < D$.

Hence an individual with ability θ decides to train if and only if the following arbitrage condition is satisfied:

$$\int_t^{t+D} e^{-\int_t^s r(\tau)} w_L(s)\, ds < \int_{t+T}^{t+D} e^{-\int_t^s r(\tau)} \max(\theta - \gamma, 0)\, w_H(s)\, ds, \quad (2)$$

with $0 < \gamma < 1/2$. Notice that an individual with ability $\theta > \gamma$ is pustulatedly able to accumulate human capital $(\theta - \gamma)$ after training, while an individual with ability lower than γ (i.e. $\theta < \gamma$) never gets any skill from schooling.

Like Dinopoulos and Segerstrom (1999) I will focus on the balanced growth path (BGP) analysis, in which all variables grow at a constant rate and w_L, w_H, and c_θ are all constant, furthermore $r(s) = \rho$ at all dates. Considering equation

[7] As in Dinopoulos and Segerstrom (1999), it is easy to show that the parameters above must satisfy $\delta = \frac{n}{e^{nD}-1}$ and $\beta = \frac{ne^{nD}}{e^{nD}-1}$, in order for the number of births at time t to match the number of deaths at $t + D$.

(2) with the equality, the ability threshold θ_0 is easily obtained which renders individual indifferent to becoming skilled or to remaining unskilled for all her life. Hence the individual will train if and only if her ability is higher than

$$\theta_0 = \left[\left(1 - e^{-\rho D}\right) / \left(e^{-\rho T} - e^{-\rho D}\right)\right] \frac{w_L}{w_H} + \gamma = \sigma \frac{w_L}{w_H} + \gamma. \tag{3a}$$

where $\sigma \equiv \left(1 - e^{-\rho D}\right) / \left(e^{-\rho T} - e^{-\rho D}\right) > 1$. An individual with ability $\theta > \theta_0$ will decide to train and will accumulate quantity $(\theta - \gamma)$ of human capital. The higher the individual ability, the higher the accumulated human capital and the higher is the total amount of wages earned by the individual. Budget constraint in equation (1) implies that an individual with higher ability will benefit from a higher value of consumption flow.

Following the same steps as Dinopoulos and Segerstrom (1999) the reader can easily verify that the supply of unskilled labor at time t is

$$L(t) = \theta_0 N(t) = \left(\sigma \frac{w_L}{w_H} + \gamma\right) N(t) \tag{4}$$

and the supply of skilled labor at time t is

$$H(t) = (\theta_0 + 1 - 2\gamma)(1 - \theta_0) \frac{\phi}{2} N(t), \tag{5}$$

where $\phi = \left(e^{n(D-T)}\right) / \left(e^{nD} - 1\right) < 1$. Along the BGP the growth rate of both unskilled and skilled labor is equal to n.

3.2 Manufacturing

Final goods and services are sold by monopolistic firms which are protected by a patent law for the production of their products. Government provides an institutional protection for innovations represented by an infinitely lived patent granted to the researcher who introduces a novel, useful, and non-obvious improvement of any existing product line. This allows the R&D firm to gain monopolistic rents for all of the real duration of the patent, because - as usual in Schumpeterian growth models with vertical innovation (see e.g. Grossman and Helpman, 1991, and Aghion and Howitt, 1992, 1998) - the incumbent monopolist can be replaced by the next innovator in the same product line. Manufacturing firms hire unskilled workers to produce any consumption good/service $\omega \in [0, 1]$ of the second-best quality under a one-to-one constant returns to scale (CRS) technology, described by a simple unit cost function w_L.[8]

As mentioned above - in a Schumpeterian quality ladder framework - the next quality of a given good/service is the offspring of the R&D performed by challenger researchers in order to replace the incumbent producer, and to gain monopolistic rents. During the temporary monopoly the top quality patent-holder can sell her product at a price higher than the marginal cost, but the

[8] It could be possible to introduce a technical coefficient $\eta > 0$ common to all industry lines without altering the qualitative results of the model. I prefer to adopt a one-to-one technology as in Grossman and Helpman (1991) for the sake of simplicity.

existence of a competitive economy-wide fringe sets a ceiling to it equal to the economy-wide lowest unit cost of the previous quality product. Because of both non-drastic innovation and instantaneous Bertrand competition along any product line ω, the limit price $p\left(j^{\max}\left(\omega,t\right),\omega,t\right)$ of every top quality product is equal to λw_L, where $\lambda > 1$ is the common quality jump in all industry lines. The unskilled labor is chosen as the *numeraire* of the economy, so that $w_L = 1$.

Let us depart from the current literature and let us assume that patents are not perfectly enforceable when a new competitor enters the market by producing a 'partial' imitation of the state-of-the-art along any product line $\omega \in [0,1]$. For partial imitation I mean a product with a lower quality than the existing state-of-the-art, but with a higher quality than the previous incumbent producer. Therefore, between two given quality-rungs - say j and $j+1$ where λ^{j+1} is the highest quality produced by the incumbent monopolist - a firm is able to produce a lagged-quality good which may infringe the claim of a patent based on lagging breadth. Patent law is defined by the legislative sector, but as indicated in the Introduction, Courts can determine the degree of protection against partial imitations.[9]

The potential threat that a new competitor may enter the market and sell an imperfect imitation of the latest vintage, without resulting with certainty in a legal infringement, will induce the quality leader to lower the limit price in order to serve the whole market. This continue to hold also in the event that the patent infringement could be recognized by the Courts. Indeed, the mere existence for the quality leader of a positive probability of losing the patent suit when it is imperfectly imitated, will induce it to lower the limit price.[10] The existence of a cumulative probability distribution function (c.d.f.) $F\left[\varepsilon_\omega;\mu_\omega\right]$ representing the leader's probability of winning a patent dispute against partial imitators in a product line ω is assumed, with the corresponding positive and continuously differentiable density function $f\left[\varepsilon_\omega;\mu_\omega\right]$.[11]

Hence $F\left[\varepsilon_\omega;\mu_\omega\right] > 0$, which is assumed constant over time, is the plaintiff's probability of winning the legal fight against partial imitators, and $1 - F\left[\varepsilon_\omega;\mu_\omega\right]$

[9] See, among others, Scotchmer (2004) for evidence about the role of Courts in the U.S. in determining the effective breadth and level of patent protection.

[10] See the Introduction about the empirical evidence that a 95% of filed patent suits are settled before trials.

[11] Along any product line, it is assumed the existence of a continuum degree of partial imitations. This means that there exists a continuum of quality levels along any rung of the ladder. The fixed and constant quality jump size λ can be interpreted as a minimum innovation size required to obtain a patent grant, that is it can be read as a patentability requirement size. Because this paper focuses on lagging breadth and on an imperfect patent protection against imitations, the role of other patent instruments - such as patentability requirement and leading breadth - is not analyzed (see O'Donoghue and Zweimuller, 2004). The parameter μ_ω proxies the degree of tolerance by which Courts judge patent suits concerning lagging breadth infringement in industry line $\omega \in [0,1]$. μ_ω order the c.d.f. in a first order stochastic dominance sense, i.e. a higher μ_ω indicates a tighter IPR enforcement by Courts against partial imitations. Then, a higher μ_ω implies a higher c.d.f. for any value of the limit price. To fix ideas, if $\mu_\omega^{'} > \mu_\omega$, it will be $F\left[\varepsilon_\omega;\mu_\omega^{'}\right] > F\left[\varepsilon_\omega;\mu_\omega\right]$, or assuming continuously differentiable c.d.f. and continuous values for IPR tightness μ_ω, it is $\frac{\partial F[\varepsilon_\omega;\mu_\omega]}{\partial \mu_\omega} > 0$. The interpretation of the parameter μ_ω stems from the use of the doctrine of disclosure and enablement.

is the probability of losing the legal fight against a partial imitation. The parameter ε_ω concerns the degree of a partial imitation, that is it indicates the quality lag between the state-of-the-art product and the partial imitation. The closer is the imitation to the top quality product, i.e. the higher is the degree of imitation ε_ω, the higher is the probability to win a legal suit by the top quality patent-holder. When $\varepsilon_\omega = 1$, a partial imitation coincides with the preceding vintage, whereas when $\varepsilon_\omega = \lambda$, a partial imitation perfectly replicates the top quality of any brand. In the remaining parts of the paper $\varepsilon_\omega \in]1, \lambda]$ is assumed, with $F[\lambda; \mu_\omega] = 1$, so that the plaintiff's probability of winning a legal fight against partial imitators is equal to 1, whenever partial imitation perfectly replicates that state-of-the-art.[12] Instead, as the degree of imitation approximates the second best quality product along the quality ladder, i.e. $\varepsilon_\omega \to 1$, the higher is the probability of winning the patent suit by a partial imitator, with $F[1; \mu_\omega] = 0$. Therefore, I will concentrate to the extent in which partial imitations constitute an inferior quality version of the state-of-the-art, and a better quality version of the second best quality product along the same industry line.

Let us analyze the optimal imitator's behavior. A partial imitator maximizes her expected profit flow from an imitation of the state-of-the-art quality product. Because patent allows a perfect and complete knowledge spillover, to manufacture a partial imitation it is assumed that no R&D effort is needed.[13]

On the other side, the partial imitator takes the R&D firm's decision as given when she solves her maximization problem.

Therefore, each partial imitator chooses the optimal degree of imitation, i.e. chooses ε_ω, by maximizing

$$\left[\int_1^\lambda \pi_i(\omega, t)\, d\bar{F}(\varepsilon_\omega; \mu_\omega)\right] \tag{6}$$

where $\pi_i(\omega, t)$ is the profit flow from a partial imitation of the state-of-the-art quality product, $\bar{F}(\varepsilon_\omega; \mu_\omega)$ is the tail of the c.d.f. $F(\varepsilon_\omega; \mu_\omega)$, i.e. $[1 - F(\varepsilon_\omega; \mu_\omega)]$, that represents the probability of winning a patent suit by a partial imitator.[14] Because the profit flow is increasing in the degree of imitation ε_ω, while the tail $\bar{F}(\varepsilon_\omega; \mu_\omega)$ is decreasing in ε_ω, the imitator's objective

[12] In this case the imitation perfectly replicates the state-of-the-art, i.e. all the claims of a patent are infringed.

[13] In the U.S. patent law, the enablement requirement states that the patent application must describe the invention clearly enough so that somebody "with ordinary skill in the art can make and use ti without undue experimentation". Indeed, in this model skilled workforce is able to generate new ideas, and manufacturing firms buying the patent are assumed to be able to implement the idea described in the patent application. Therefore, also a partial imitator is assumed able to implement an easier version of a new patented idea, and she can at most incur in an implementation cost. Indeed, all the analysis and results also hold by assuming that a fixed implementation cost must be incurred for a successful imitation. This assumption is very similar to Acemoglu, Aghion, and Zilibotti (2006). Indeed, these authors consider the existence of a fringe of firms that can "steal" the leader technology and produce the same good by incurring in a higher production cost.

[14] In the case of the existence of a fixed implemantation cost, the maximization problem becomes $\left[\int_1^\lambda \pi_i(\omega, t)\, d\bar{F}(\varepsilon_\omega; \mu_\omega)\right] - c$, where c is a fixed implementation cost.

function is assumed to be concave in ε_ω, so that at least a solution to the maximization problem exists. Yet, the solution could not be unique. Let us define $\varepsilon^*_{\omega i}$ to be a solution i to the above maximization problem. Therefore, because free entry in the imitation activity is assumed, in equilibrium it must be $\pi^*_i(\omega,t)\,\bar{F}(\varepsilon^*_{\omega i};\mu_\omega)=0$.

As usual in ladder quality model with non-drastic innovations, the producer of the top quality, i.e. the leader, adopts a limit pricing strategy. In such a framework, the limit price that allows the leader to serve the whole market will be $(\lambda/\bar{\varepsilon}_\omega)$, where $\bar{\varepsilon}_\omega \equiv \min\{\varepsilon^*_{\omega i}\}$, i.e. it is the lowest optimal degree of imitation satisfying the free entry condition in the imitation activity (see the Appendix A for a proof). Because this limit price strategy allows the quality leader to serve the whole market, it implies that a partial imitator is always out of the market.[15]

In the light of the instantaneous household preferences, the consumer θ demand quantity for each product $\omega \in [0,1]$ is

$$q(\omega,t) \equiv \int_0^1 \frac{c_\theta N(t)}{p(\omega,t)}d\theta = \frac{N(t)}{(\lambda/\bar{\varepsilon}_\omega)}\int_0^1 c_\theta d\theta = \frac{cN(t)}{(\lambda/\bar{\varepsilon}_\omega)} \tag{7}$$

where $c \equiv \int_0^1 c_\theta d\theta$ indicates the per-capita consumption fraction of any product $\omega \in [0,1]$, and $q(\omega,t)$ is the quantity sold by each monopolist. It follows that the stream of expected profit flows accruing to the monopolist which manufactures the state-of-the-art quality product ω will be equal to:

$$\pi(\omega,t) = q(\omega,t)\left[(\lambda/\bar{\varepsilon})_\omega - 1\right] = cN(t)\left[1-\frac{1}{(\lambda/\bar{\varepsilon}_\omega)}\right] \tag{8}$$

3.3 R&D Sector

In a vertical R&D framework, the state-of-the-art incumbent producer is challenged by outsider R&D firms that employ skilled workers in order to introduce a better quality version of any existing good and service. As usual in ladder quality models à la Grossman and Helpman (1991) and Aghion and Howitt (1992, 1998) Arrow's effect is at work.

Every R&D firm i can produce an instantaneous Poisson arrival rate of innovation $I_i(\omega,t)$ in the product line $\omega \in [0,1]$ it targets by using a CRS technology described by unit cost function $bw_H\ X(\omega,t)$, with $b>0$ common to all industries, and $X(\omega,t)>0$ measuring the degree of complexity in the invention of the next quality product in industry $\omega \in [0,1]$. The returns to

[15] This reflects the empirical evidence indicated in the Introduction showing that almost 95% of filed patent suits are settled before trial, and the imitator is out of the market. The lower profit flow a top quality leader gains because of a lower limit price can therefore be interpreted as a proxy for settling a legal dispute. Notice that, the quality leader always prefers to be a monopolist rather than to share the market with a partial imitator. This is due to the constant marginal cost of production. Moreover, because of free entry in the imitation activity, new imitators would enter the market whenever the quality leader and a partial imitator sharing the market gained a positive profit.

R&D investment are independently distributed across firms, across industries, and over time. Therefore the industry-wide arrival rate of innovation in industry ω at time t is $I(\omega,t)=\sum_i I_i(\omega,t)$, which represents the aggregate summation of the Poisson arrival rate of innovation produced by all R&D firms targeting product $\omega \in [0,1]$.

The technological complexity argument as indexed by factor $X(\cdot)$ was introduced into R&D-based endogenous growth models after Charles Jones' (1995) empirical criticism of the first strand of Schumpeterian endogenous growth models, which showed scale effects on per-capita output growth rate. The PEG specification for the law of motion of the technological complexity index suggested by Dinopoulos and Segerstrom (1999) is adopted:[16]

$$X(\omega,t)=kN(t) \tag{9}$$

with $k>0$, thereby formalizing the idea that it is more difficult to introduce a new product in a more crowded market. In the present framework with quality improving consumer goods and services, the per-capita growth rate of the economy is represented by the increase in the representative consumer utility level.

Let us $v(\omega,t)$ to denote the expected stock market value of a successful R&D firm in industry ω at time t. Because each leader is targeted by R&D firms that try to discover the next top quality product, the shareholder suffers a loss $v(\omega,t)$ with probability $I(\omega,t)\,dt$. Whereas the event of no innovation occurs with probability $[1-I(\omega,t)\,dt]$. Over a time interval dt, the shareholder of a stock issued by a successful R&D firm receives a dividend $\pi(\omega,t)\,dt$ and the value of the firm appreciates by $dv(\omega,t)=\dot{v}(\omega,t)\,dt$. Since the stock market is assumed perfectly efficient, the expected rate of return of a stock issued by a successful R&D firm must be equal to the riskless rate of return r:

$$rdt=\frac{\dot{v}(\omega,t)}{v(\omega,t)}\left[1-I(\omega,t)\,dt\right]dt-\frac{v(\omega,t)-0}{v(\omega,t)}\left[I(\omega,t)\right]dt+\frac{\pi(\omega,t)}{v(\omega,t)}dt$$

Taking the limits as $dt\to 0$, the following condition for the expected discounted value of the firm producing good ω is obtained:

$$v(\omega,t)=\frac{\pi(\omega,t)}{\rho+I(\omega,t)-\frac{\dot{v}(\omega,t)}{v(\omega,t)}}=\frac{cN(t)\left[1-\frac{1}{(\lambda/\bar{\varepsilon})_{\omega}}\right]}{\rho+I(\omega,s)-\frac{\dot{v}(\omega,t)}{v(\omega,t)}} \tag{10}$$

[16] Jones (1995) showed that the strong scale effect prediction of the first strand of Schumpeterian endogenous growth models - i.e. the per capita output growth rate was predicted to positively depend on the population level - is at odds with the empirical evidence. The acronym "PEG" used to remove the strong scale effect refers to the "permanent effects on growth" of policy measures. This is distinguished by the law of motion for technological complexity defined as "TEG", which produces "temporary effects on growth" of policy measures. Here a dilution method is used to eliminate the strong scale effect. See Madsen (2008) to empirically motivate this choice.

where $r = \rho$ since the analysis refers to the BGP equilibrium. As in Dinopoulos and Segerstrom (1999), along the BGP the per-capita variables all grow at the same rate, it follows that $\frac{\dot{v}(\omega,t)}{v(\omega,t)} = \frac{\dot{\pi}(\omega,t)}{\pi(\omega,t)} = n$. Hence the discounted expected profit value can be written as

$$v(\omega,t) = \frac{cN(t)\left[1 - \frac{1}{(\lambda/\bar{\varepsilon}_\omega)}\right]}{\rho + I(\omega,t) - n} \tag{11}$$

Each R&D firm targeting product $\omega \in [0,1]$ chooses its R&D intensity to maximize $v(\omega,t)\, I_i(\omega,t) - bX(\omega,t)\, w_H I_i(\omega,t)$. The R&D sector is characterized by a perfectly competitive environment, with free entry and exit and CRS technology. This implies that for any industry line $\omega \in [0,1]$ targeted by positive R&D the following no-arbitrage condition holds:

$$v(\omega,t) = \frac{cN(t)\left[1 - \frac{1}{(\lambda/\bar{\varepsilon}_\omega)}\right]}{\rho + I(\omega,t) - n} = bX(\omega,t)\, w_H \tag{12}$$

where an interior solution to the above maximization problem by any R&D firm i exists, that is the innovative effort is positive and finite in any product line. Since in each industry line there can be a different degree of tolerance of partial imitations, there will exist an innovation Poisson arrival rate structure in the economy. The no-arbitrage equation (12) implies that the industry lines with a higher limit price, and hence gaining higher profit flows, will have a correspondingly higher creative destruction discount factor, i.e. they will have a higher Poisson arrival rate of innovation $I(\omega,t)$. Indeed, higher profit flows in an industry line spur more innovative effort in it until the higher increasing creative destruction exactly offsets the higher rents in the same industry line. This process will continue and in equilibrium equation (12) will be satisfied for each variety $\omega \in [0,1]$.

4 Balanced Growth Path

Given the economic environment described in section 2, the general equilibrium implications of the economy are analyzed. In this framework, the BGP is the only rational expectation equilibrium of the model.

Since each final good monopolist employs unskilled labor economy-wide to accomplish manufacturing, the unskilled market clearing equilibrium condition is:

$$N(t)\,\theta_0 = \int_0^1 q_\omega d\omega = \frac{cN(t)}{\lambda}\int_0^1 \bar{\varepsilon}_\omega d\omega = \frac{cN(t)}{\lambda}\digamma \tag{13}$$

where $\digamma \equiv \int_0^1 \bar{\varepsilon}_\omega d\omega$. Therefore:

$$c = \lambda\theta_0/\digamma, \tag{14}$$

By considering both equations (12) and (7), it is possible to obtain the expected stock market value for any good/service as

$$\frac{cN(t)\left[1-(\bar{\varepsilon}_{\omega}/\lambda)\right]}{\rho+I(\omega,t)-n}=bw_{H}X(\omega,t), \tag{15}$$

which - since $w_H = \frac{\sigma}{\theta_0-\gamma}$ and (14) holds - can be rewritten as

$$\frac{\lambda\theta_0}{F}\left[1-(\bar{\varepsilon}_{\omega}/\lambda)\right]=\frac{b\sigma k}{(\theta_0-\gamma)}\left[\rho+I(\omega,t)-n\right], \tag{16}$$

where $k=\frac{X(\omega,t)}{N(t)}$ denotes the population-adjusted degrees of complexity for any product $\omega\in[0,1]$.

As for unskilled labor, it is possible to obtain the market clearing equilibrium condition for the skilled labor force. By using both equations (5) and the CRS technology production function of innovating firms, the skilled labor market equilibrium condition is

$$(\theta_0+1-2\gamma)(1-\theta_0)\frac{\phi}{2}=bk\int_0^1 I(\omega,t)\,d\omega=bkI^* \tag{17}$$

where $I^*(t)\equiv\int_0^1 I(\omega,t)\,d\omega$, and the PEG formulation for the increasing technological complexity has been considered.

Then, considering both equations (17) and (18), it is possible to state the following

Proposition 1 *Under PEG specification, along the balanced growth path a threshold ability exists such that $\theta_0>\gamma$. Along the balanced growth path θ_0 is an increasing function of $\bar{\varepsilon}_{\omega}$.*

Proof. *See the Appendix A.* ■

Therefore, proposition 1 implies that Courts' degree of tolerance on the legal fight between the incumbent monopolist and the partial imitator affects the incentive for human capital accumulation.

5 Imperfect Intellectual Property Rights Enforcement

In this Section, the effect of a tighter/softer IPR protection against partial imitations is analyzed. Within this framework, a tighter IPR enforcement shows up in the limit price of the top-quality leader. A natural measure to capture a lower IPR enforcement consists in a change in the c.d.f. $F[\varepsilon_{\omega};\mu_{\omega}]$ such that a higher value $\bar{\varepsilon}_{\omega}$ is obtained. Indeed, with this change the leader is forced to reduce her limit price, and still supply the highest quality along the same product line. Therefore, a higher value of $\bar{\varepsilon}_{\omega}$ can be interpreted as a lower

IPR enforcement along any industry line ω, while a lower value of $\bar{\varepsilon}_\omega$ can be interpreted as a tighter IPR enforcement.[17]

In order to analyze these effects let us consider proposition 1. It is immediately evident that a tighter IPR enforcement in the economy - as represented by a reduction in $\bar{\varepsilon}_\omega$ along each product line - spurs human capital accumulation, i.e. θ_0 is lower along the new BGP. The unskilled labor market clearing condition immediately implies a reduction in the total quantity of goods and services that can be produced with the existing unskilled labor force.

To analyze the effect of a tighter IPR protection on both innovation rate and inequality, let us consider the no-arbitrage equation (12) for any good/service ω that, by substituting in it equation (14), can be written as:

$$v(\omega, t) = \frac{\frac{\lambda\theta_0}{F}\left[1 - (\bar{\varepsilon}_\omega/\lambda)\right]}{\rho + I(\omega, t) - n} = bkw_H. \quad (18)$$

It is noteworthy to recall that - because the unskilled labor is the numeraire - w_H is a measure of the skill premium, and not the absolute level of the skill wage. Along the new BGP with a tighter IPR enforcement, the top quality leader along any product line ω obtains a higher instantaneous profit flow because of a higher limit price. This implies a higher incentive to innovate, and therefore a positive demand excess in the skill labor market that raises the skill premium. The increase in the skill premium induces a higher fraction of the population to accumulate human capital through schooling. Indeed, along the new BGP a lower threshold ability θ_0 is associated to a higher skill premium w_H (see equation 3). Along the new BGP with a tighter IPR enforcement, both the supply and demand of skilled population are higher as well as wage inequality between an unskilled and skilled population does. The final effect on the aggregate Poisson arrival rate of innovation along the new BGP is easily obtained. An economy with a tighter IPR protection has a higher mass of skill resources. Because the mass of industry lines is constant, the Poisson arrival rate of innovation will be correspondingly higher. Notice that these results hold even if a tighter IPR protection for the patent-holder is guaranteed along any subset of the existing product lines. To fix ideas, if Courts decide for a tighter IPR protection along some product lines, the same economic mechanisms and results along the new BGP are obtained.

Therefore, the following can be stated:

Proposition 2 *More enforceable IPR - as represented by a reduction in the parameter $\bar{\varepsilon}_\omega$ - along any subset of the existing product lines determines: 1) a higher human capital accumulation; 2) a higher skill premium; 3) a reduction in*

[17] In this framework, a tighter IPR enforcement also implies a higher limit price, i.e. a higher markup. As indicated in the Introduction, chemicals are a technological field with a tighter IPR enforcement than other industry lines such as computers and communications. The model's prediction of a higher markup in industries with a tighter IPR enforcement is in line with Norrbin's (1993) findings that chemicals is an industry line with one of the highest markup.

the total quantity of consumed goods and services; 4) an increase in the aggregate rate of innovation and in the per capita output growth rate.

Proof. *See the Appendixes A and B* ■

When a tighter IPR enforcement exists in only a subset of the existing industry lines - with a non zero measure - the aggregate Poisson arrival rate of innovation will be higher along the new BGP. Yet, in such a case some further changes between industry lines arise. In particular, whenever a tighter IPR enforcement in any subset $\eta \in]0,1]$ with a non zero measure of the existing industry lines allows the increase in the instantaneous profit flow in the same subset η to outweigh the increase in the skill premium - i.e. condition (B8) holds along the new BGP - the Poisson arrival rate of innovation will be higher in each industry line $\omega \in [0,1]$. Yet, that industries with a tighter IPR enforcement - i.e. a subset η - will have a lower increase in their Poisson arrival rate of innovation than other industry lines. On the contrary, whenever condition (B8) does not hold, the Poisson arrival rate of innovation will be lower along that subset of industries with a tighter IPR enforcement, and all the increase in the aggregate rate on innovation will stay on the shoulders of that industry lines with an unchanged IPR enforcement. Therefore, a tighter IPR enforcement in a subset of the existing industries generates a positive externality - in terms of the Poisson arrival rate of innovation - on the complementary subset of industry lines. These results stem from a change in the demand and supply composition between industry lines. Indeed, in a subset of industry lines with an unchanged IPR enforcement, the limit price is unchanged - i.e. $\left[\frac{\lambda}{\bar{\varepsilon}_\omega} - 1\right]$ is unchanged - so that an increase in the per capita profit flow is due to an increase in the sold quantity. Therefore, along a new BGP with a tighter IPR enforcement in only a subset of the existing industries, that industries with a tighter IPR enforcement will have a lower quantity produced and consumed, and all other industry lines will have a higher quantity produced and consumed.

Finally, an economy with a tighter IPR enforcement has a higher consumption level inequality between unskilled and skilled population. Furthermore, because in such a case the total production is lower, the unskilled population suffers of a lower per capita consumption level.

6 Conclusions

The empirical evidence shows that both the Courts and the patent law determine the effective IPR protection for a patent-holder. In practice patent law - and in general the IPR enforcement - does not guarantee a perfect and complete protection for the patent holder. Indeed, the empirical evidence shows the existence of patent suits and the openness of legal disputes due to patent infringement, and in particular it shows that there exists the possibility for outsiders to create and to market an imperfect imitation of the state-of-the-art. This paper considers the existence of an imperfect IPR enforcement for a patent-holder, and it evaluates the effect of such an institutional setting on the long-run economic performance of an economy.

To this aim, a quality ladder growth model in the spirit of Dinopoulos and Segerstrom (1999) is adopted. There exist different routes for intellectual protection. In this paper, this happens through the use of the doctrine of disclosure and enablement interpretation whereby infringement of the lagging breadth are at the core of disputes. The economic framework considers an endogenous choice of any partial imitator concerning the degree of its imitations. In doing so, an imitator faces a positive probability to lose a legal fight against the top quality patent-holder, being this probability higher as the imitating product is more similar to the state-of-the-art. It is shown that the mere existence of such a threat forces the patent-holder to reduce the monopolistic limit price, and this strategy allows the top quality patent-holder to serve the whole market along any product line.

The results show that when Courts' decisions tend to favor the patent-holder against potential imitations, the model generates a positive effect on human capital accumulation. On the other side, a tighter IPR enforcement determines a higher skill premium and exacerbates inequality between an unskilled and skilled population. Moreover, because of a higher skill premium and a lower production, the consumption level inequality between an unskilled and skilled population is higher, and the unskilled population suffers of a lower average consumption level than an economy with a softer IPR enforcement. Finally, both the aggregate innovation rate and the per capita output growth rate are higher with a tighter IPR enforcement. When the tightness of the IPR enforcement is heterogeneous between technological fields and industries, a positive IPR enforcement externality is found whereby the innovation rate is higher in the relatively less protected industries than in fields with a tighter IPR protection. Therefore, a main result concerns the role of patent enforcement in each technological field and industry line in affecting wage inequality and innovation rates for the whole economy. In particular, it suffices a tighter/softer patent enforcement in some technological fields - rather than in all industry lines of a country - to produce highly different economic performance for an economy.

Appendix A

In this appendix, the optimal limit pricing strategy of the top quality leader along any product line ω is obtained. Along a ladder quality of any product line each vintage of the product is a perfect substitute of any other, once a quality adjustment is considered. This depends on linear preferences - quality adjusted - for all the different versions of the same good/service along a product line. As usual in ladder quality model, it is assumed that consumers only buy the product vintage with the highest quality-price ratio. Because of both non-drastic innovations and instantaneous Bertrand competition, the quality leader adopts a limit pricing strategy and will serve the whole market. Let us consider two successive rungs along a ladder, say $j+1$ and j, along a line ω. The state-of-the-art has quality λ^{j+1}, while the second best vintage has quality λ^{j}. Therefore, a partial imitation of size ε_ω of the top quality product has a quality $\lambda^j\varepsilon_\omega$. The quality ratio between the top quality $j+1$, and a partial imitation of size ε_ω will be $\lambda/\varepsilon_\omega$. Let us consider a top quality partial imitator that has solved her maximization problem (6). To be concrete, let us suppose that only two values $\varepsilon_{\omega 1}$ and $\varepsilon_{\omega 2}$ satisfy the first order condition, with $\varepsilon_{\omega 1} < \varepsilon_{\omega 2}$. We also know that - because of logarithmic preferences with unit demand elasticity and of a constant marginal cost of production - the instantaneous profit flow is an increasing function of the limit price. Let us consider a partial imitation of size $\varepsilon_{\omega 1}$. A limit pricing strategy implies that the quality-price ratio for the state-of-the-art and for the partial imitation will respectively be $\frac{\lambda^{j+1}}{\lambda/\varepsilon_{\omega 1}}$ and $\frac{\lambda^j\varepsilon_{\omega 1}}{\varepsilon_{\omega 1}}$. Each consumers will compare these quality-price ratio, with $\frac{\lambda^{j+1}}{\lambda/\varepsilon_{\omega 1}} > \frac{\lambda^j\varepsilon_{\omega 1}}{\varepsilon_{\omega 1}}$, because $\varepsilon_{\omega 1} > 1$. Therefore, with this limit pricing strategy each consumer will buy the top quality product at price $\lambda/\varepsilon_{\omega 1}$. No consumer would buy an imitation of size $\varepsilon_{\omega 1}$ at a price higher than the limit price $\varepsilon_{\omega 1}$. Any partial imitator that reduces her price below $\varepsilon_{\omega 1}$ would incur in lost, and the first order condition would be violated. Indeed, the instantaneous profit flow is an increasing function of the limit price, and the Courts decide about infringement by considering the size of imitation, i.e. the probability to lose a legal fight is $F(\varepsilon_{\omega 1};\mu_\omega)$ whatever is the imitators' limit price. This holds even if a partial imitator would offer the product for free, because the Patent law consider also an offer as an infringing product. Therefore any partial imitator of size $\varepsilon_{\omega 1}$ does not find profitable to sell at a price lower than $\varepsilon_{\omega 1}$. Let us consider a partial imitation of size $\varepsilon_{\omega 2} > \varepsilon_{\omega 1}$. A limit pricing strategy implies that each consumers will compare $\frac{\lambda^{j+1}}{\lambda/\varepsilon_{\omega 2}}$ and $\frac{\lambda^j\varepsilon_{\omega 2}}{\varepsilon_{\omega 2}}$, with $\frac{\lambda^{j}}{\lambda/\varepsilon_{\omega 2}} > \frac{\lambda^j\varepsilon_{\omega 2}}{\varepsilon_{\omega 2}}$ because $\varepsilon_{\omega 2} > 1$, where $\frac{\lambda^{j+1}}{\lambda/\varepsilon_{\omega 2}}$ is the quality-price ratio of the top quality leader, and $\frac{\lambda^j\varepsilon_{\omega 2}}{\varepsilon_{\omega 2}}$ is the quality-price ratio of the partial imitator. With this limit pricing strategy each consumer will buy the top quality product at price $\lambda/\varepsilon_{\omega 2}$. Again, any partial imitator that reduced her price below $\varepsilon_{\omega 2}$ would incur in lost, and the first order condition would be violated. Indeed, the instantaneous profit flow is an increasing function of the limit price, and the Courts decide about infringement by considering the size of imitation, i.e. the probability to lose a legal fight is $F(\varepsilon_{\omega 2};\mu_\omega)$ whatever is the imitators' limit price. Therefore any partial imitator of size $\varepsilon_{\omega 2}$ does not find it profitable to sell at a price lower than $\varepsilon_{\omega 2}$. Yet, in such a case, the top-quality leader could charge a price $\lambda/\varepsilon_{\omega 1}$, also for a partial imitation of size $\varepsilon_{\omega 2}$. In this case any consumer would compare $\frac{\lambda^{j+1}}{\lambda/\varepsilon_{\omega 1}}$ and $\frac{\lambda^{j}\varepsilon_{\omega 2}}{\varepsilon_{\omega 2}}$, with $\frac{\lambda^{j+1}}{\lambda/\varepsilon_{\omega 1}} > \frac{\lambda^{j}\varepsilon_{\omega 2}}{\varepsilon_{\omega 2}}$, because $\varepsilon_{\omega 1} > 1$. The top quality leader would gain higher profits by charging a limit price $\lambda/\varepsilon_{\omega 1}$, because $\lambda/\varepsilon_{\omega 1} > \lambda/\varepsilon_{\omega 2}$, and the instantaneous profit flow is an increasing function of the limit price. Therefore, by admitting the existence of multiple solutions $\{\varepsilon^*_{\omega i}\}$ to the partial imitator maximization problem (6) along the product line ω, the optimal limit pricing strategy of the top quality leader is to charge $(\lambda/\bar{\varepsilon}_\omega)$, where $\bar{\varepsilon}_\omega \equiv \min\{\varepsilon^*_{\omega i}\}$. Notice that consumers buy the quality level λ^{j+1}, whatever the limit price of the top-quality leader is. Q.E.D.

Appendix B

1. In the first part of this appendix the threshold ability θ_0 along the BGP is obtained, then the existence of a negative relation between threshold ability θ_0 and $\bar{\varepsilon}_\omega$ is proved.

Solving equation (17) for $I(\omega, t)$ and integrating over ω's field it is possible to obtain the economy-wide Poisson arrival rate of innovation:

$$I^*(t) = \int_0^1 I(\omega, t)\, d\omega = \frac{\theta_0(\theta_0 - \gamma)}{\sigma b k}\left[\frac{\lambda}{F} - 1\right] - (\rho - n) \tag{B1}$$

Hence equation (18) can be written as:

$$(\theta_0 + 1 - 2\gamma)(1 - \theta_0)\frac{\phi}{2} \equiv \frac{\theta_0(\theta_0 - \gamma)}{\sigma}\left[\frac{\lambda}{F} - 1\right] - bk(\rho - n) \tag{B2}$$

The left hand side (LHS) of equation (B2) is a strictly concave quadratic polynomial with roots $(2\gamma - 1)$ and 1, and the right hand side (RHS) of equation (B2) is a strictly convex quadratic polynomial with two real roots, one negative and one positive, where the positive roots is

$$\theta_0 = \frac{1}{2}\left\{\gamma + \sqrt{\gamma^2 + 4\frac{\sigma b k(\rho - n)}{\left[\frac{\lambda}{F} - 1\right]}}\right\} \in (\gamma, 1) \tag{B3}$$

if the stated parameter restrictions are satisfied. Therefore, there exists one, and only one, real and positive solution $\theta_0 \in (\gamma, 1)$.

The equation (B2) can be written as

$$G(\cdot) = -(\theta_0 + 1 - 2\gamma)(1 - \theta_0)\frac{\phi}{2} + \frac{\theta_0(\theta_0 - \gamma)}{\sigma}\left[\frac{\lambda}{F} - 1\right] - bk(\rho - n) \tag{B2bis}$$

Using the Implicit Function Theorem for equation (B2bis) the inverse relationship between threshold ability θ_0 and a tighter IPR enforcement as represented by a lower value of $\bar{\varepsilon}_\omega$ is obtained,

$$\frac{\partial\theta_0}{\partial\bar{\varepsilon}_\omega} = -\frac{\frac{\partial G(\cdot)}{\partial\bar{\varepsilon}_\omega}}{\frac{\partial G(\cdot)}{\partial\theta_0}} > 0 \tag{B4}$$

Inequality (B4) derives from the fact that
$\frac{\partial G(\cdot)}{\partial \theta_0} = (\theta_0 - \gamma)\,\phi + \frac{(2\theta_0 - \gamma)}{\sigma}\left[\frac{\lambda}{F} - 1\right] > 0$, and from the fact that the sign of $\frac{\partial G(\cdot)}{\partial \bar{\varepsilon}_\omega} = -\frac{\lambda\theta_0(\theta_0-\gamma)}{\sigma}\frac{1}{F^2} < 0$. Q.E.D.

2. In this part, the effect of an increase in $\bar{\varepsilon}_\omega$ on the industry-wide Poisson arrival rate of innovation and on long-run economic growth is studied. By considering both equations (3) and (B4), it is easy to verify that

$$\frac{\partial w_H}{\partial \bar{\varepsilon}_\omega} = -\frac{\sigma \frac{\partial \theta_0}{\partial \bar{\varepsilon}_\omega}}{(\theta_0 - \gamma)^2} < 0 \tag{B5}$$

Let us consider the per-capita instantaneous profit flow along any industry line ω along the BGP rewritten as $\frac{\lambda\theta_0}{F\frac{\lambda}{\bar{\varepsilon}_\omega}}\left[\frac{\lambda}{\bar{\varepsilon}_\omega} - 1\right] = \frac{\lambda\theta_0}{F} - \frac{\theta_0\bar{\varepsilon}_\omega}{F}$. Now, a comparison between an infinitesimal change in the per-capita profit flow along an industry line with a tighter IPR enforcement, and an infinitesimal change in the per-capita profit flow along an industry line with a constant IPR enforcement is considered. Let us define the change in per-capita profit flow in any industry line ω with a tighter IPR enforcement as $\Delta\pi_T$. This change is

$$\Delta\pi_T \equiv \frac{\partial \pi_T}{\partial \bar{\varepsilon}_\omega} = \frac{(\lambda - \bar{\varepsilon}_\omega)\left[\frac{\partial \theta_0}{\partial \bar{\varepsilon}_\omega}F - \frac{\partial F}{\partial \bar{\varepsilon}_\omega}\theta_0\right] - F\theta_0}{F^2} \tag{B6}$$

while, let us define the change in per-capita profit flow in any industry line ω that remains with the same IPR enforcement as $\Delta\pi_N$. This change is

$$\Delta\pi_N \equiv \frac{\partial \pi_N}{\partial \bar{\varepsilon}_\omega} = \frac{(\lambda - \bar{\varepsilon}_\omega)\left[\frac{\partial \theta_0}{\partial \bar{\varepsilon}_\omega}F - \frac{\partial F}{\partial \bar{\varepsilon}_\omega}\theta_0\right]}{F^2} \tag{B7}$$

Therefore, from equations (B6) and (B7) it follows $\Delta\pi_N = \Delta\pi_T + \frac{\theta_0}{F}$, which implies $\Delta\pi_T < \Delta\pi_N$. Because the instantaneous profit flow is an increasing function of the limit price $\frac{\lambda}{\bar{\varepsilon}_\omega}$, a tighter IPR enforcement along any industry line ω implies a higher instantaneous profit flow along the same industry line, i.e. $\Delta\pi_T > 0$ for a lower $\bar{\varepsilon}_\omega$. Therefore, in the light of both equations (B6) and (B7) also $\Delta\pi_N > 0$, and this change is larger than $\Delta\pi_T$. Notice that, along the industry lines with an unchanged IPR enforcement, the limit price is unchanged - i.e. $\left[\frac{\lambda}{\bar{\varepsilon}_\omega} - 1\right]$ is unchanged - so that an increase in the per capita profit flow is due to an increase in the sold quantity. Therefore, along a new BGP with a tighter IPR enforcement in only a subset of the existing industries, that industries with a tighter IPR enforcement will have a lower quantity produced and consumed, and all other industry lines will have a higher quantity produced and consumed. Finally, whenever condition

$$|\Delta\pi_T| > \left|\frac{\partial w_H}{\partial \bar{\varepsilon}_\omega}\right| \tag{B8}$$

is satisfied, a tighter IPR enforcement along a subset of the existing industry lines determines a higher Poisson arrival rate of innovation along all industry lines, especially along that industries with an unchanged IPR enforcement. Q.E.D.

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