Vertical integration and the licensing of innovation with a fixed fee or a royalty *

Stéphane Lemarié †
UMR GAEL
INRA – Université Pierre Mendès-France

October 2008

Abstract

In this paper, we analyse a situation where a patent holder is considered as an upstream firm who can license its innovation to some downstream companies that compete on a final market with differentiated products. Licensing contract may be based either on a royalty or a fixed fee. The analysis is made in a context where the choice of vertical integration by the patent holder is endogenous.

We show that a license based on a royalty works better with vertical integration, so that the patent holder always prefers to integrate vertically if he can only apply a royalty based license. When the two types of license can be used, the patent holder chooses to integrate vertically if and only if he chooses a royalty rather than a fixed fee. In other terms, a royalty is used by an integrated firm and a fixed fee is used by a non integrated firm. The effect of vertical integration on the social surplus can be either positive or negative.

Keywords: Licensing, Innovation, Vertical Integration

JEL: D45, L22, L42, O31, O32

*I thank Stefan Ambec, Eric Avenel, Stéphane Caprice, Vianney Dequiedt, Marion Desquilbet, Vincent Réquillart, and Shyama Ramani for their very helpful comments on the previous versions of the paper. The appendix of this document provides the detailed proofs with Cournot competition. The appendix with the detailed proofs with Bertrand competition can be provided upon request.

†UMR GAEL, INRA – Université Pierre Mendès-France, BP 47, 38040 Grenoble Cedex 9, France. email: lemarie@grenoble.inra.fr; phone: (33) 4 76 82 78 84; fax: (33) 4 76 82 54 55.

Introduction

It is generally recognised that licensing can represent a large share of the firms' profit, especially in the technology intensive sectors. This fact is associated with the development of markets for technologies [Arora et al., 2002], where the upstream actors develop some new technologies, and the downstream actors incorporate them into new products or processes in order to improve their competitive position on the final market. In such a setting, a patent holder may choose either to be only an upstream technology provider (or an outsider innovator), or to vertically integrate some downstream activities (becoming then, an insider or an incumbent innovator). With no vertical integration, the patent holder earns profit only from licensing its innovation. With vertical integration, the patent holder can either license its innovation or foreclose its downstream competitor(s). The aim of this paper is to analyse the interest of a patent holder for vertical integration and the welfare impact of this choice.

The comparison of different payment structure (royalty, fixed fee, auction, two-part tariff) is a major topic in the literature on the licensing of innovation. The first contributions were more particularly concerned with the interest of a royalty *vs* a fixed fee or an auction mechanism. On the one hand, the theoretical analysis highlights the interest of a fixed fee (or auction) compared to a royalty, in a context where the innovator is not vertically integrated [Kamien and Tauman, 1986, Kamien et al., 1992]. On the other hand, the empirical literature shows that royalty based licenses are rather frequent [Rostocker, 1983, Caves et al., 1983]. Several recent theoretical contributions enable to understand this contradiction. Muto [1992] and Caballero-Sanz et al. [2002] show that a royalty can be preferred to a fixed fee when considering a market with product differentiation. It is shown also that the innovator can prefer a royalty when it is vertically integrated with one downstream licensee [Wang, 1998, Wang and Yang, 1999, Wang, 2002, Kamien and Tauman, 2002]. At last, the innovator can prefer a royalty in a context with uncertainty or information asymmetry on the innovation quality [Bousquet et al., 1998, Gallini and Wright, 1990].

Some recent theoretical analysis have also introduced more complex payment structure where a royalty can be combined with a fixed fee or an auction [Erutku and Richelle, 2000, Faulí-Oller and Sandonís, 2003, Sandonís and Faulí-Oller, 2006, Sen and Tauman, 2003]. Such contract is generally preferred by the innovator because he can then extract a larger share and sometime all the surplus generated by the innovation.[1] However, the empirical literature shows that the use of a simple payment structure is rather frequent:

- Rostocker [1983] analyses the licensing practices of 37 US firms, and show that

[1] This result can be derived from the fact that corner solutions with only a royalty of a fixed fee generally do not appear at the equilibrium.

licensing contracts based only on a royalty or a fixed fee occur in 52% of the cases, while two-part tariff occur in 46% of the cases.

- Macho-Stadler et al. [1996] analyse a sample of 241 licensing contracts between Spanish firms (licensees) and foreign companies (licensor), for a large range of sector at the beginning of the 1990's. In this sample, more than 80% of the contract are based only on a royalty or a fixed fee, while only 10% of the contract are based on two-part tariff.

- Jensen and Thursby [2001] study the licensing practices of 62 universities in the US between 1991 and 1995. Their survey shows that the universities frequently combine two types of payment, and in particular a fixed fee and a royalty.

It is difficult to exclude, *a priori* some type of licensing contract. Hence, the analysis of the licensing strategy should be done with a large range of contract.

All the theoretical contributions mentioned before, except one [Sandonís and Faulí-Oller, 2006], consider that vertical integration is exogenous. The aim of this paper is to analyse the optimal licensing strategy when the patent holder can choose whether or not to vertically integrate in a preliminary stage. By doing so it is possible to better understand to what extend vertical merger can be explained by motivations related to the licensing of intellectual property rights, and to analyse the welfare impact of such mergers.

The analysis is based here on a simple model with no uncertainty and no information asymmetry. Competition on the final market occurs between two firms selling one differentiated product each. Both Cournot and Bertrand competition are considered. One of these two firms is the subsidiary of the upstream patent holder in the case with vertical integration. Licensing contract may be based either on a royalty or a fixed fee. This model is identical to the one studied by Muto [1992] with no vertical integration and Wang and Yang [1999] and Wang [2002] with vertical integration. Hence the results from these three articles will be synthesised in this paper, and completed with an analysis of the interest for vertical integration This model is also identical to the one used by Sandonís and Faulí-Oller [2006], except that these authors consider only licenses based on two-part tariff. The combined two papers provide interesting conclusions about vertical integration and licensing with a large range of payment structures.

The first important result of this paper is that *the patent holder always integrates vertically when only a royalty based license can be used*. Two arguments explain this result. First, a royalty is a poor instrument for extracting the net profit let to the independent downstream licensee. This problem is lessened with vertical integration

because the licensee faces a more competitive environment. Second, vertical integration increases the industry profit when the royalty is high enough because the double margin is then less important (it concerns one non integrated firm instead of two).

The second important result is that, when vertical integration is endogenous and either a royalty or a fixed fee based license can be used, *integrated company chooses a royalty while non-integrated company chooses a fixed fee*. Vertical integration has no effect on the patent holder profit with a fixed fee (except in some particular cases which are not relevant here). Hence, the patent holder chooses to vertically integrate only when a royalty is preferred to a fixed fee.

The paper is organised as follow. The model is presented in the section 1. We then analyse the interest for vertical integration with each type of licensing: royalty only (section 2), fixed fee only (section 3), and a royalty or a fixed fee (section 4). The analysis is structured this way because all the properties with the two types of license are based on properties when only one type of license can be used. Extensions with alternative specification of the demand and alternative licensing contracts are addressed in the section 5.

1 The model

The model is first presented in a configuration with no vertical integration. The (minor) modifications made with vertical integration are presented after.

We consider a final market with two differentiated and competing products. Each product is produced and sold by a specific firm ($i = 1$ or $i = 2$). Two versions of each product can be produced depending on whether or not it incorporates an innovation. In practice this innovation can be considered as an improvement of one characteristic that increases the willingness to pay of the consumers for the product. It is supposed that each firm can produce and sell only one version of its product. The dummy variable θ_i is used to indicate whether or not the product sold by the firm i incorporates ($\theta_i = 1$) or not ($\theta_i = 0$) the innovation. The property right of the innovation is owned by a third actor, called either the patent holder or the upstream firm. In contrast, the two firms 1 and 2 are called the downstream firms.

The inverse demand function on the final market is defined as follow: [2]

$$p_i = a + \theta_i \delta - b q_i - \lambda b q_j \tag{1}$$

[2]This demand function can be derived from the utility function of a representative consumer defined as follow (see Singh and Vives [1984] for more details):

$$U(q_1, q_2) = (a + \theta_1 \delta) q_1 + (a + \theta_2 \delta) q_2 - \frac{b}{2}(q_1^2 + 2\lambda q_1 q_2 + q_2^2)$$

a is the highest willingness to pay for the product when it does not incorporate the innovation ($a > 0$), and δ is the additional willingness to pay for the product when it incorporates the innovation ($\delta > 0$). b reflects the own price elasticity of the demand ($b > 0$). λ reflects the degree of substituability between the product ($\lambda \in [0, 1]$). The case where $\lambda = 0$ corresponds to a maximum differentiation of the product: the two products are sold on two different and independent final markets. Conversely, when $\lambda = 1$ the two products are perfect substitutes.

The innovation is patent-protected and can be incorporated in one product if a license contract is signed with the patent holder. Two types of licensing contract are considered in this paper: (i) a royalty based license where the licensee pays w to the patent holder for each unit he sells, (ii) a fixed fee based license, where the licensee pays F to the patent holder whatever the quantity he sells. The contracts are supposed to be public. The marginal production cost of the firm i is $c + \theta_i w$. The marginal production cost of the upstream firm is equal to zero because it only sells an access to some intellectual property.

The interactions between the actors are structured in three stages. First the patent holder decides the type of licensing contract and the corresponding variable (w with a royalty, F with a fixed fee). The same contract is proposed to the two downstream firms. Second, each of the downstream firm decides whether he accepts or refuses the license contract (decision variable θ_i). Third, competition occurs on the final market with either Cournot or Bertrand competition [3].

An alternative configuration will be considered, where the patent holder and one of the two downstream firms are vertically integrated. The subscript v is then used to refer to the vertically integrated firm and the subscript s is used to refer to the independent downstream firm. The subsidiary of the vertically integrated company has a free access to the patent. The three stages presented before are still valid with vertical integration. The only difference is that, at the stage 2, we are concerned with the decision of only one downstream firm (the firm s).

The vertical integration choice is done before the three basic stages described before. We suppose that there is no cost associated with vertical integration, so that the patent holder chooses to integrate vertically one licensee if it leads to an increase of their joint profit.

The resolution is made by backward induction. The three basic stages will be solved first and the choice concerning vertical integration will be analysed after.

[3] Note that the results would be identical with a model where the innovation enables to decrease the production cost from c to $c - \delta$, but does not affect the product characteristics. This alternative modelling is the one used by most of the contribution on licensing [c.f. for example Wang, 2002].

Note also that the stage 1 resolution indirectly leads the patent holder to choose a certain level of access restriction to the innovation. Access restriction is defined as follow: there is no access restriction when the innovation is incorporated in the two products ($\theta_1 = \theta_2 = 1$), a partial access restriction when the innovation is incorporated in only one product ($(\theta_1, \theta_2) \in \{(1,0),(0,1)\}$), and a complete access restriction when no product incorporate the innovation ($\theta_1 = \theta_2 = 0$).

2 Analysis with a royalty based license

The resolution is presented first in details with Cournot competition (section 2.1 to 2.3). The differences that appears with Bertrand competition are briefly presented after (section 2.4). The characteristics of the stage 3 to stage 1 subgames equilibrium are briefly presented here because this analysis has already been done in several contributions.

2.1 Stages 3 and 2 subgames

- Stage 3 subgame equilibrium. $\pi_i(w, \theta_i, \theta_j)$ is the the gross profit of firm i at the stage 3 equilibrium. A monopoly equilibrium can appear if $\theta_i \neq \theta_j$ and if δ is high enough: the monopoly earns then $\pi_i(w, 1, 0) > 0$ and the competitor earns $\pi_i(w, 0, 1) = 0$. Conversely, if $\theta_i = \theta_j$ or if δ is small enough, the stage 3 equilibrium is a duopoly. With vertical integration, the equilibrium gross profit are $\pi_v(w, \theta_s)$ and $\pi_s(w, \theta_s)$. The firm v can be in a monopoly position if $\theta_s = 0$ and if δ is high enough or if $\theta_s = 1$ and if w is very high. Otherwise, the stage 3 equilibrium is a duopoly. The expressions of the stage 3 equilibrium are presented in the appendix A.

Thereafter, the analysis we will be based to a large extend on the industry profit: the stage 3 equilibrium industry profit is $\Pi_{NI}(w, \theta_1, \theta_2)$ with no vertical integration, and $\Pi_I(w, \theta_s)$ with vertical integration.

- Stage 2 subgame equilibrium. With no vertical integration, a downstream firm accepts the license if the royalty is lower than a certain threshold level $\hat{w}_{NI}$. Note that this level is identical whatever the choice of the competitor. Hence, two equilibrium are possible at the stage 2: the two downstream firms accept the license if $w \leq \hat{w}_{NI}$ and reject it if $w > \hat{w}_{NI}$. Partial access restriction never appears in this case because the best response of each firm does not depend on the choice of the competitor. With vertical integration, the firm s accept the license if the royalty is lower than $\hat{w}_I$. With Cournot competition, we have $\hat{w}_I = \hat{w}_{NI} = \delta$.

2.2 Stage 1 subgame

- Optimal royalty level with no access restriction. The optimal royalty level with no vertical integration is noted w^*_{NI}. If the innovation is important enough ($\delta \geq \delta_{NI}$) the maximisation of the patent holder profit leads to an *unconstrained* level $\tilde{w}_{NI}$ lower than $\hat{w}_{NI}$. Conversely (if $\delta < \delta_{NI}$), the optimal royalty level is constrained to be equal to $\hat{w}_{NI}$.

Similarly, with vertical integration, the patent holder chooses an optimal unconstrained royalty level ($w^*_I = \tilde{w}_I$) if the innovation is important enough ($\delta \geq \delta_I$), and a constrained level ($w^*_I = \hat{w}_I$) otherwise. Note that $\delta_I < \delta_{NI}$, so that the optimal royalty level is more often unconstrained with vertical integration. Note also that, with Cournot, the optimal royalty level is lower with vertical integration, but the difference is very small ($w^*_I / w^*_{NI} \in [0.97, 1]$).

- Interest for access restriction.

Lemma 1 *With a royalty based license, the patent holder prefers not to restrict the access at the stage 1 equilibrium, with either vertical or no vertical integration.*

With vertical integration, this result has already been established in the literature [Wang, 2002, Wang and Yang, 1999] with process innovation. This result is also valid here because the characteristics of the stage 3 equilibrium are similar.

With no vertical integration, the interest for no access restriction is straightforward: partial access restriction cannot appear at the stage 2 equilibrium, and the patent holder earns no profit with complete access restriction. One could wonder however if partial access restriction could be interesting with a modified version of the model such that partial access restriction can appear at the equilibrium. We can suppose for example that the patent holder can credibly commit to sign only one license agreements at the beginning of the stage 1, before choosing the level of the royalty. However, it can be shown that the lemma 1 still holds, even with this modified version of the model [4].

2.3 The effects of vertical integration

The objective of this section is to show that a royalty based license works better with vertical integration, so that the patent holder always prefers to integrate vertically.

Lemma 2 *With a royalty based license and no access restriction, vertical integration leads to a decrease of the licensee profit:*

$$\pi_s(w^*_I, 1) < \pi_i(w^*_{NI}, 1, 1)$$

[4] The proof of this result is available upon request.

The two products sold on the final market incorporate the innovation. Hence, the difference of the licensee profit is caused by the difference of production cost. With vertical integration the licensee (firm s) faces a competitor with a lower production cost than with no vertical integration, because the competitor has a free access to the innovation in the former case, while he pays a royalty in the last case. Rigorously, one have to observe also that the production cost of the licensee is lower with vertical integration, because of the lower royalty level. However this last effect is negligible compared to the first one (see appendix B for a detailed proof).

Proposition 1 *With a royalty based license, the patent holder always prefer to integrate vertically.*

Vertical integration leads to an increase of the social surplus (Cournot specific).

The patent holder has an interest for vertical integration if:

$$\begin{aligned} & \Pi_I(w_I^*,1) - \pi_s(w_I^*,1) > [\Pi_{NI}(w_{NI}^*,1,1) - 2\pi_i(w_{NI}^*,1,1)] + \pi_i(w_{NI}^*,1,1) \\ \Leftrightarrow \quad & \Pi_I(w_I^*,1) - \Pi_{NI}(w_{NI}^*,1,1) > \pi_s(w_I^*,1) - \pi_i(w_{NI}^*,1,1) \end{aligned}$$

The term on the right hand side of the first inequality is the joint profit between the patent holder (in square brackets) and one licensee. We have just shown (lemma 2), that the term on the right hand side of the second inequality is negative.

The detailed proof of this inequality is given in appendix C. If the patent holder can apply an unconstrained royalty level ($\delta \geq \delta_{NI}$) vertical integration leads to an increase of the industry profit (i.e. the left hand side of the second inequality is positive). If the patent holder has to apply a constrained royalty level, its gain from decreasing the licensee profit with vertical integration is always greater than the variation of industry profit.

Vertical integration leads to an increase of the social surplus because the prices on the final market are lower with vertical integration. Two results explain this property. First, with a given royalty level, the prices are lower with vertical integration ($p_v(w,1) < p_s(w,1) < p_i(w,1,1)$) because the competition is then more intense. Second, the optimal royalty level is lower with vertical integration ($w_I^* \leq w_{NI}^*$). Note however that this last result is specific to the Cournot competition.

2.4 Analysis with Bertrand competition on the final market

With Bertrand competition, three types of equilibrium appear at the stage 3: a duopoly and a monopoly equilibrium like with Cournot competition, and, in-between, a constrained monopoly. In the constrained monopoly, only one firm has positive sales, but he is constrained to apply a limit price lower than the monopoly price in order to keep the competitor out of the market.

All the lemma and proposition established with Cournot competition are still valid with Bertrand competition, except one: vertical integration can lead to a surplus decrease if the products are close substitutes and the innovation is minor. This result comes from the fact that the maximum royalty level defined from the stage 2 resolution can be much higher with vertical integration ($\hat{w}_I > \hat{w}_{NI}$) especially when the product are close substitutes. As a consequence, when the optimal royalty level with no vertical integration is constrained (i.e. when δ is small), the optimal royalty can be much higher with vertical integration. Such a high royalty level can leads to higher prices on the final market, and to a surplus decrease.

3 Analysis with a fixed fee based license

The stage 3 resolution made with a royalty is valid with a fixed fee: the gross profit with a fixed fee is equal to the gross profit with a royalty when $w = 0$. All the analysis made in this section is valid both with Cournot and Bertrand competition on the final market.

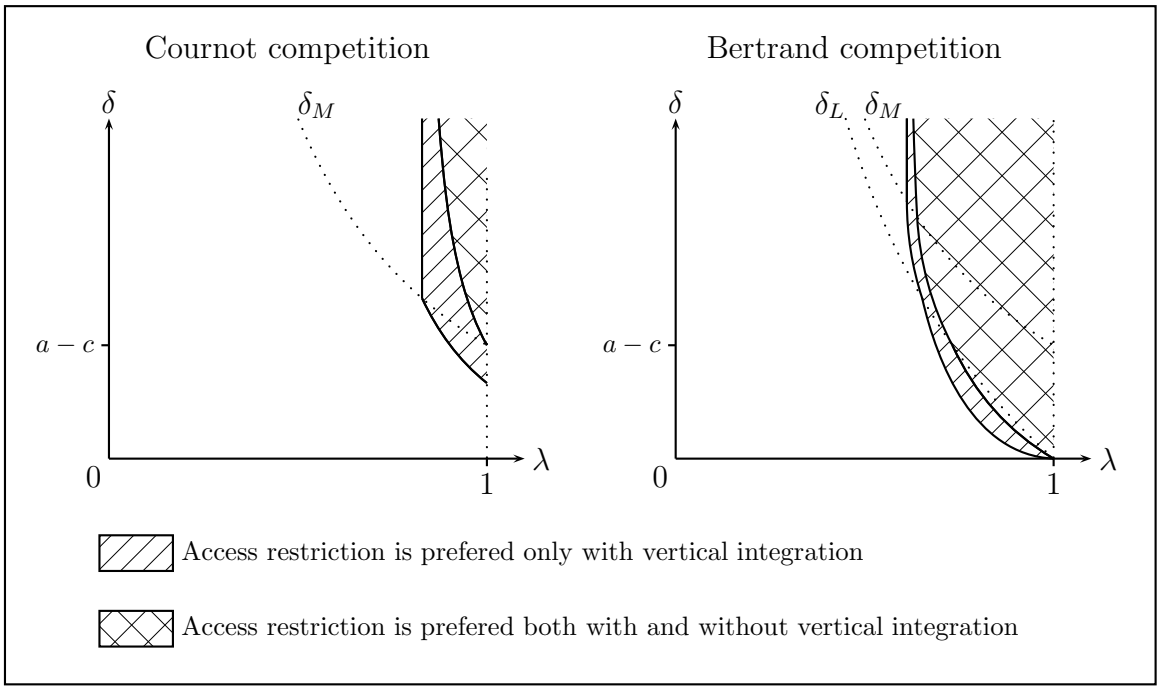
3.1 Stages 2 and 1 subgames

- Optimal fixed fee with a given access restriction With vertical integration, the net profit of the firm s is $\pi_s(0,1) - F$ if it accepts the license and $\pi_s(0,0)$ if it rejects it. Hence, the firm s accepts the license at the stage 2 if $F \leq F_I$ with $F_I = \pi_s(0,1) - \pi_s(0,0)$. At the stage 1, the patent holder has no interest to define a fixed fee lower than F_I if he chooses not to restrict the access. Hence, the patent holder chooses F_I with no access restriction and $F > F_I$ with partial access restriction. The net profit of the firm s is $\pi_s(0,0)$ in both cases.

With no vertical integration, the choice of one potential licensee at the stage 2 depends on the choice of its competitor. If j accepts the license, then i also accepts the license if $F \leq F_2$ with $F_2 = \pi_i(0,1,1) - \pi_i(0,0,1)$. If j reject the license, then i accepts the license if $F \leq F_1$ with $F_1 = \pi_i(0,1,0) - \pi_i(0,0,0)$. After observing that F_2 is always lower than F_1 (cf. appendix D), we can conclude that three equilibrium are possible at the stage 2. If the fixed fee is very low ($F < F_2$) the two firms accept the license at the equilibrium because there best response is to accept the license whatever the choice of the competitor. If the fixed fee is intermediate ($F \in [F_2, F_1]$), only one firm accepts the license at the equilibrium because the best response of i is to make the converse choice of the competitor. Finally, if the fixed fee is high ($F > F_1$), the two firms reject the license at the equilibrium.

Compared to the royalty based license, a new equilibrium with partial access restriction can appear with no vertical integration. This equilibrium exists because

Figure 1: Interest of the patent holder for partial access restriction with a fixed fee



there is a range of value for the fee ($F \in [F_2, F_1]$) where the best response of each downstream firm is to make the converse choice compared to the competitor.

At the stage 1, if the the patent holder does not restrict the access, he chooses a fixed fee equal to F_2 and the net profit let to the licensee is $\pi_i(0, 0, 1)$. If the patent holder restricts (partially) the access, he chooses a fixed fee equal to F_1, the net profit let to the licensee is $\pi_i(0, 0, 0)$ and the net profit of the excluded firm is $\pi_i(0, 0, 1)$.

- Interest for access restriction

Lemma 3 *With a fixed fee license, the patent holder prefers to restrict (partially) the access if the innovation is important enough and if the products are close enough substitutes.*

Wang [2002] already defined the conditions that leads the patent holder to restrict the access in the case with vertical integration. We extend here this result to the case with no vertical integration (see appendix E). We also provide a representation of the zone where the patent holder chooses to restrict the access (figure 1).

With vertical integration, the patent holder prefers to restrict the access if it leads to an increase of the industry profit:

$$\Pi_I(0,0) - \pi_s(0,0) > \Pi_I(0,1) - \pi_s(0,0) \quad \Leftrightarrow \quad \Pi_I(0,0) > \Pi_I(0,1)$$

With no vertical integration, the condition is different but the increase of the industry profit is still a necessary condition.

The qualitative result behind the lemma 3 can be understood by discussing the effect of access restriction on the industry profit. On the one hand, access restriction has a positive effect on the industry profit because it leads to an increase of the market power of the unique licensee. On the other hand, the industry profit is negatively affected by access restriction because the innovation is incorporated in one product only instead of two. Finally, with close enough substitutes and important enough innovation, access restriction is preferred because its positive effect on the industry profit increases while the negative effect decreases.

3.2 Effects of vertical integration

- Effect of vertical integration on the industry profit

Lemma 4 *With a fixed fee and a given restriction of the access to the innovation, the profit of the industry is identical with or without vertical integration:*

$$\Pi_I(0,\theta) = \Pi_{NI}(0,1,\theta) = \Pi_{NI}(0,\theta,1)$$

$\theta = 0$ *with partial access restriction, and* $\theta = 1$ *with no access restriction.*

With a given access restriction, the fixed fee does not affect the production cost of the two downstream companies, and consequently does not affect the equilibrium on the final market. Fixed fee leads only to a transfer among firms but does not affect the industry profit. Formally we have $\pi_v(0,\theta) = \pi_i(0,1,\theta)$, $\pi_s(0,\theta) = \pi_i(0,\theta,1)$, and $\Pi_{NI}(0,\theta,1) = \pi_i(0,\theta,1) + \pi_i(0,1,\theta) = \pi_s(0,\theta) + \pi_v(0,\theta) = \Pi_I(0,\theta)$

- Effect of vertical integration on the interest for access restriction

Proposition 2 *With a fixed fee, if the patent holder prefers to restrict (partially) the access with no vertical integration, then he prefers also to restrict the access with vertical integration.*

Using the lemma 4, we can observe that access restriction leads to the same variation of industry profit with or without vertical integration ($\Pi_{NI}(0,1,0) - \Pi_{NI}(0,1,1) = \Pi_I(0,0) - \Pi_I(0,1)$).

With vertical integration, the patent holder restricts the access if and only if it leads to an increase of the industry profit. With no vertical integration, access restriction leads to an increase of the net profit of the downstream firm (from $2\pi_i(0,0,1)$ to $\pi_i(0,0,1)+\pi_i(0,0,0)$). Hence, if access restriction is preferred with no vertical integration, it means that it leads to a strictly positive variation of industry profit, and consequently access restriction is also preferred with vertical integration. In other words, access restriction is more often preferred with vertical integration because it does not lead to an increase of the licensee profit as it does with no vertical integration.

This proposition explains why, in the figure 1, there is a set of parameters value for λ and δ such that the patent holder has an interest for a partial access restriction with vertical integration but no interest for access restriction with no vertical integration.

- Interest of the patent holder for vertical integration

Proposition 3 *With a fixed fee, there is an interest for vertical integration only if it enables the patent holder to move from a strategy with no access restriction (with no vertical integration) to a strategy with access restriction (with vertical integration).*

Vertical integration leads to a decrease of the social surplus.

If vertical integration does not affect the choice of access restriction by the patent holder, then it does not affect the equilibrium on the final market. As a consequence, the patent holder has no interest for vertical integration because it does not affect the joint profit of the patent holder and one licensee[5].

Suppose now that the parameters are such that there is an interest for access restriction only with vertical integration. With vertical integration we have then:

$$\Pi_I(0,0)-\pi_s(0,0)>\Pi_I(0,1)-\pi_s(0,0)$$

Using the lemma 4, this inequality is equivalent to:

$$\Pi_I(0,0)-\pi_s(0,0)>[\Pi_{NI}(0,1,1)-2\pi_i(0,0,1)]+\pi_i(0,0,1)$$

The term on the right hand side is the joint profit of the patent holder (between square bracket) and one licensee with no access restriction. Finally, the patent holder prefers

[5]Formally, if no access restriction if preferred in both cases, we have:

$$\underbrace{\Pi_{NI}(0,1,1)-2\pi_i(0,0,1)}_{\text{patent holder profit with no V.I.}}+\underbrace{\pi_i(0,0,1)}_{\text{licensee net profit with no V.I.}}=\underbrace{\Pi_I(0,1)-\pi_s(0,0)}_{\text{patent holder profit with V.I.}}$$

and, if access restriction is preferred in both cases, we have:

$$\underbrace{\Pi_{NI}(0,1,0)-\pi_i(0,0,1)-\pi_i(0,0,0)}_{\text{patent holder profit with no V.I.}}+\underbrace{\pi_i(0,0,0)}_{\text{licensee net profit with no V.I.}}=\underbrace{\Pi_I(0,0)-\pi_s(0,0)}_{\text{patent holder profit with V.I.}}$$

to integrate vertically in this case when there is an interest for access restriction only with vertical integration

With a fixed fee, the surplus is maximum with no access restriction because the innovation is then integrated in the two products and the marginal cost of the licensee is minimal. Vertical integration leads to a decrease of the social surplus because it is chosen only if it enables to move from no access restriction (i.e. maximum surplus) to access restriction (i.e. non-maximum surplus).

4 Analysis with a royalty or a fixed fee based license

We consider here that the patent holder chooses first whether of not to integrate vertically and then the best type of license between a royalty and a fixed fee. Thereafter, the three basic stages of the model take place.

Vertical integration can lead the patent holder to change its licensing strategy. The table 1 provides a useful guideline of the multiple comparison that needs to be made between the licensing strategies with or without vertical integration. We will first show (section 4.1) that some comparison can be eliminated because a certain licensing strategy with vertical integration implies a certain licensing strategy with no vertical integration. We will then analyse the interest of the patent holder for vertical integration and show that this choice determines the type of license (section 4.2).

All the analysis presented here is valid both with Cournot and Bertrand competition on the final market.

Table 1: Synthesis of the patent holder best strategy with a royalty or a fixed fee

Licensing strategy with vertical integration	Licensing strategy with no vertical integration	
	Royalty	Fixed fee
Royalty	(a) Integration	(b) Integration
Fixed fee	(c) $\emptyset$	(d) No integration

4.1 Interest for fixed fee *vs* royalty

Lemma 5 *With no access restriction, the net profit of a licensee is lower with a fixed fee compared to a royalty, either with or without vertical integration:*

$$\pi_s(0,0) \leq \pi_s(w_I^*, 1) \qquad \textit{and} \qquad \pi_i(0,0,1) \leq \pi_i(w_{NI}^*, 1, 1)$$

This lemma states that the fixed fee is more efficient than the royalty in extracting the rent of the licensee. The detailed proof is given in the appendix F.

Proposition 4 *If the patent holder prefers to apply a fixed fee rather than a royalty with vertical integration, then he prefers to apply a fixed fee rather than a royalty with no vertical integration, and he prefers not to restrict the access both with and without vertical integration.*

This property is the indirect consequence of the fact that a royalty license works better with vertical integration, while the fixed fee performs equivalently both with and without vertical integration. Hence it is not possible to find a case where the royalty would be preferred to fixed fee with no vertical and integration and the converse with vertical integration.

It can first be observed that, because of the the lemma 1, the patent holder has no interest to restrict the access with vertical integration and a fixed fee (i.e. cases (c) and (d)). [6]

Suppose that we are in the cases (c) or (d): the patent holder prefers to use a fixed fee (and no access restriction) rather than a royalty with vertical integration:

$$\Pi_I(0,1) - \pi_s(0,0) > \Pi_I(w_I^*,1) - \pi_s(w_I^*,1)$$

Using the proposition 1, we know that the profit with a royalty and vertical integration (right hand side) is greater to the joint profit of the patent holder and one licensee with a royalty and no vertical integration. Consequently, we have:

$$\Pi_I(0,1) - \pi_s(0,0) > \Pi_{NI}(w_{NI}^*,1,1) - \pi_i(w_{NI}^*,1,1)$$

Using the lemma 4, we can replace the term on the left hand side by an equivalent term with no vertical integration:

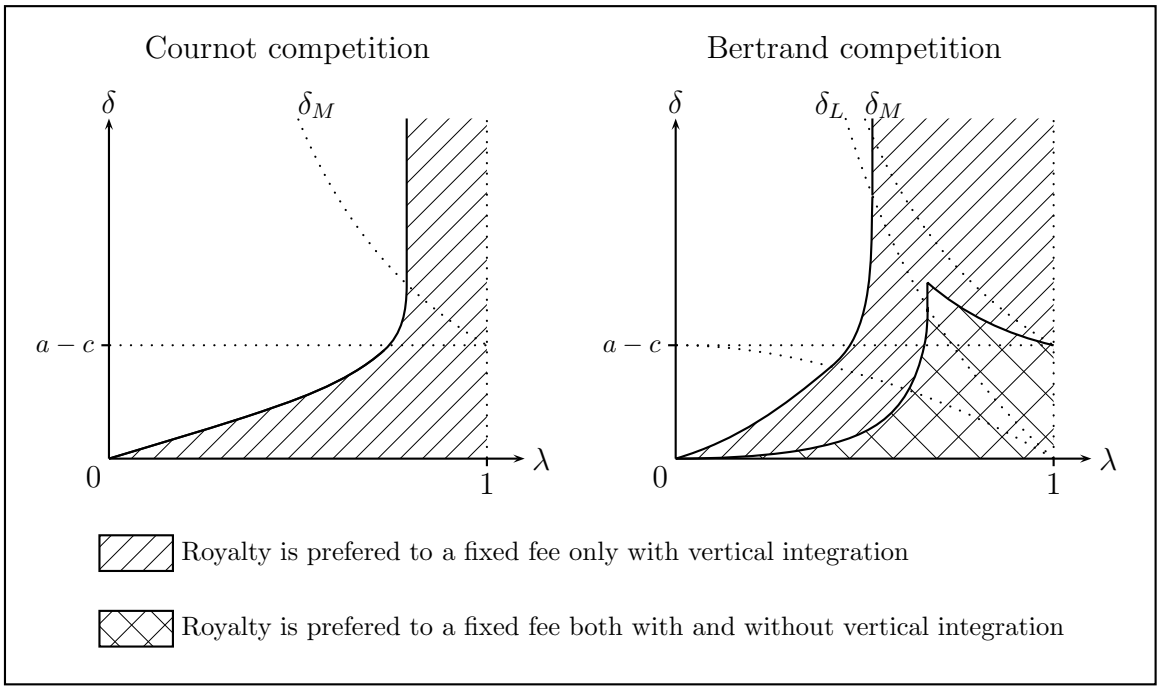
$$\Pi_{NI}(0,1,1) - \pi_i(0,0,1) > \Pi_{NI}(w_{NI}^*,1,1) - \pi_i(w_{NI}^*,1,1)$$

We then subtract $\pi_i(0,0,1)$ on both sides and use the lemma 5 to get:

$$\begin{aligned} \Pi_{NI}(0,1,1) - 2\pi_i(0,0,1) &> \Pi_{NI}(w_{NI}^*,1,1) - \pi_i(w_{NI}^*,1,1) - \pi_i(0,0,1) \\ &> \Pi_{NI}(w_{NI}^*,1,1) - 2\pi_i(w_{NI}^*,1,1) \end{aligned}$$

[6] The lemma 1 states that the patent holder prefers not to restrict the access with a royalty. With vertical integration and access restriction, the patent holder profit is the same with a royalty or a fixed fee ($\Pi_I(0,0) - \pi_s(0,0)$) simply because there is no contract. Hence, the strategy with a fixed fee and access restriction is dominated by the strategy with a royalty (and no access restriction).

Figure 2: Interest of the patent holder for a royalty rather than a fixed fee



This inequality implies that the patent holder prefers to use a fixed fee rather than a royalty with no vertical integration. In other terms, the case (c) is not possible.

Finally, to complete the proof, one have to show that the patent holder never restricts the access in the case (d). We have shown that the patent holder does not restrict the access with a fixed fee and vertical integration. By using the proposition 2, we can conclude that he does not restrict the access either with no vertical integration.

The figure 2 defines more precisely the zone where each type of licensing strategy is chosen by the patent holder.

4.2 Interest for vertical integration

Proposition 5 *If the patent holder can use either a fixed fee or a royalty based license, there is a net interest for vertical integration if and only if it leads the patent holder to choose a royalty based licensing contract.*

Because of the proposition 4, only three cases of the table 1 needs to be considered. In the case (a), the patent holder prefers to integrate vertically because of the

proposition 1. In the case (d), we have shown that the patent holder never restricts the access (proposition 4). By using the proposition 3, we can conclude that the patent holder prefers not to integrate. In the case (b), we can show that the patent holder prefers to integrate vertically. This result comes from the fact that the profit with a fixed fee and a given access restriction is not affected by vertical integration. Hence, if the patent holder earns more with a royalty compared to a fixed fee with vertical integration, he earns more by integrating and applying a royalty rather that not integrating and applying a fixed fee (see appendix G for more details).

The effect of vertical integration on the social surplus can be either positive or negative. With Cournot competition, the patent holder always prefers to apply a fixed fee rather than a royalty with no vertical integration. Vertical integration leads to an increase (resp. a decrease) of the social surplus if the patent holder chooses to restrict (resp. not to restrict) the access with no vertical integration (cf. appendix G).

5 Discussion

5.1 Alternative demand function

The demand function used in this paper is common in the economic literature and in most of the paper on the licencing of innovation with differentiated products. However over specification of the demand with product differentiation are possible.

It can be observed that the results with a fixed fee or a royalty (section 4) are all derived from the previous results with only one type of license (either a fixed fee or a royalty). We expect the result with a fixed fee only (section 3) to be robust with a large range of demand function[7]. The results with only a royalty based license (section 2) are specific to this demand function. However, only two particular results are important for the rest of paper: the patent holder never prefers to restrict the access (lemma 1) and always prefers to integrate vertically (proposition 1). Finally, the main results from this paper are valid with alternative demand function if these two particular results are checked.

A counter-example can be find in Caballero-Sanz et al. [2002] who consider a demand function based on a spatial differentiation model with a circular layout of the consumers. With such a model, the royalty always dominates the fixed fee and there is no interest of the patent holder for vertical integration. This results comes from the fact that the whole demand is served with any value of the royalty. Hence there is no double margin effect with such a model, and consequently no interest for

[7]The lemma 4 is robust with any demand function. The proposition 2 is valid as long as $\pi_i(0,0,1) < \pi_i(0,0,0)$, a basic and rather general property. The proposition 3 follows from the proposition 2.

vertical integration[8].

5.2 Alternative licensing contract

Auctions and two-part tariff are the two alternative types of licensing contract commonly considered in the literature on the licensing of innovation.

Consider first the case where the patent holder can sell a certain number of licenses through a first price auction mechanisms. Applying such a mechanism here makes sense only when there is at least two potential bidders (i.e. with no vertical integration). We know from the literature [Katz and Shapiro, 1986] that auction and fixed fee lead to the same profit with no access restriction if the patent holder can impose a minimum bid. With access restriction, auction is more efficient than fixed fee because it enables to reduce the net profit let to the licensee (from $\pi_i(0,0,0)$ to $\pi_i(0,0,1)$). Hence, the net profit of the downstream firms with an auction is identical with or without access restriction. Access restriction is preferred if and only if it leads to an increase of the industry profit (either with or without vertical integration).[9] Finally, vertical integration cannot lead to a change of the access restriction as it is a case with a fixed fee (cf. proposition 3). The patent holder has no interest for vertical integration if he can only use auction. If the patent holder can use a royalty or an auction, the main result of this paper still holds: the patent holder integrates vertically if and only if it leads him to choose a royalty rather than an auction.

Sandonís and Faulí-Oller [2006] consider the same model than the one used in this paper, but with two-part tariff. They show first that there is no interest for access restriction either with or without vertical integration. The optimal royalty level is a compromise between two opposite effects: on the one hand, the patent holder has an interest to decrease the royalty level in order to decrease the net profit let to the licensee; but on the other hand, the patent holder has an interest to keep the royalty level at a rather high level in order to moderate the competition on the final market and to preserve the industry profit. The more important is the innovation, the lower is the net profit of the licensee, and the easier it is for the patent holder to get a good compromise. The interest of vertical integration is to decrease the profit let

[8]Poddar and Sinha [2004] consider a similar model based on a linear rather than a circular layout of the consumers, and a linear rather than a quadratic transportation cost. Despite these differences, the results are qualitatively identical to those obtained by Caballero-Sanz et al. [2002].

[9]Formally we have:

$$\begin{aligned}\Pi_{NI}(0,1,1)-2\pi_i(0,0,1) &> \Pi_{NI}(0,1,0)-2\pi_i(0,0,1)\\ &\Leftrightarrow \Pi_{NI}(0,1,1) > \Pi_{NI}(0,1,0) \Leftrightarrow \Pi_I(0,1) > \Pi_I(0,0)\\ &\Leftrightarrow \Pi_I(0,1)-\pi_s(0,0) > \Pi_I(0,0)-\pi_s(0,0)\end{aligned}$$

to the licensee, but the drawback is that it leads to lower industry profit.[10] Finally, the patent holder prefers to integrate vertically with a small innovation and not to integrate vertically with large enough innovation. This choice leads to a decrease of the social surplus. Indeed, with moderate innovation, the surplus is greater with no vertical integration because the patent holder is then forced to apply a lower royalty level which leads to a lower prices on the final market. Conversely, with important innovation, the surplus is greater with vertical integration because it decreases the loss associated with the double margin.

Conclusion

This paper analyses the interest of an upstream patent holder to integrate vertically a downstream potential licensee in a context where either a fixed fee or a royalty based license can be used. Two main cases have to be distinguished in order to summarise the results. In the first case, only a royalty based license is possible. The patent holder has then always an interest for vertical integration and this choice generally leads to an increase of the social surplus. For the patent holder, vertical integration has two positive effects (part of the double margin is eliminated and the profit let to the licensee decreases) that always overcomes one negative effect (the more intense competition that leads to a decrease of the industry profit). The second case is when a fixed fee can be used. Vertical integration is preferred when it enables the patent holder to change qualitatively its licensing strategy: to move from no access restriction to access restriction when only a fixed fee is possible, or to move from a fixed fee to a royalty when both types of license are possible. Vertical integration leads either to an increase or a decrease of the social surplus.

These results complete those from Sandonís and Faulí-Oller [2006] who show first that vertical integration can be preferred when the innovation is small and second that such a choice leads to a decrease of the social surplus. This analysis by Sandonís and Faulí-Oller [2006] leads to the clear-cut result that all profitable vertical mergers reduce welfare (with Cournot competition). In this paper, we show that this result no longer holds if we change the type of licensing contract (with the same demand and Cournot competition). More precisely, if for some reason the patent holder cannot combine a royalty and a fixed fee, but is constrained to use only one of the two instruments, then some profitable vertical mergers increase welfare. From a competition policy perspective, this result confirms the rational for a rule of reason

[10]The profit let to the licensee is lower with vertical integration because he competes with the subsidiary of the vertically integrated company who has access to the innovation for free. Industry profit could be higher with vertical integration when the royalty level is high, but this situation never appears at the equilibrium with two-part tariff.

approach of the vertical merger cases and indicates that particular attention should be paid to the type of licensing contract used by the patent holder.

Policy recommendation can be strengthened by several development of the current model. First, one have to analyse the robustness of the result with respect to the number of downstream firms. The optimal licensing contract with an arbitrary number of downstream firms has been analysed in the literature either with or without vertical integration and with different types of payment structure[11]. However, no contribution analyses the interest of the patent holder for vertical integration in such a context. Second, when considering product innovation as we do in this paper, one should consider the case with vertical differentiation where the two types of products (the one that incorporate the innovation, and the one that does not) have positive sales. Avenel and Caprice [2003] consider such a case in a context with a manufacturer and two retailers and two part tariff contacts. Some results are qualitatively different compared to those obtained by Sandonís and Faulí-Oller [2006][12], but these differences could be explained by the use of different demand functions. Third, the dynamic effect on the incentive to invest in research should also be addressed. More precisely, what would be the effect of the vertical integration, after a first generation of innovation, on the incentive to invest in research for a second generation of innovation?

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Appendix (Cournot competition)

A Prices and quantities at the stage 3 equilibrium

- With no vertical integration. *At the duopoly equilibrium,* the quantity, price and profit are:

$$q_i(w,\theta_i,\theta_j) = \frac{(2-\lambda)(a-c)+(2\theta_i-\lambda\theta_j)(\delta-w)}{b(4-\lambda^2)}$$
$$p_i(w,\theta_i,\theta_j) = c+\theta_j w + \frac{(2-\lambda)(a-c)+(2\theta_i-\lambda\theta_j)(\delta-w)}{4-\lambda^2}$$
$$\pi_i(w,\theta_i,\theta_j) = \frac{((2-\lambda)(a-c)+(2\theta_i-\lambda\theta_j)(\delta-w))^2}{b(4-\lambda^2)^2}$$

We can check that if i does not incorporate the innovation but its competitor j does it ($\theta_i = 0$ et $\theta_j = 1$), then, at the stage 3 equilibrium, i supply a positive quantity with a positive margin if $\delta < \delta_M + w$ with $\delta_M = (2-\lambda)(a-c)/\lambda$. If $\theta_i = \theta_j = 1$, the quantities are positive if $w < a - c + \delta$. This condition is always fulfilled at the equilibrium because, otherwise, the patent holder who defines w, would make no profit.

At the monopoly equilibrium, the quantities, prices and profits are:

$$q_i(w,1,0) = \frac{a-c+\delta-w}{2b} \qquad q_i(w,0,1) = 0$$
$$p_i(w,1,0) = c+w+\frac{a-c+\delta-w}{2} \qquad p_i(w,0,1) = c$$
$$\pi_i(w,1,0) = \frac{(a-c+\delta-w)^2}{4b} \qquad \pi_i(w,0,1) = 0$$

This case appears only if $\delta \geq \delta_M + w$. The quantity sold by i is positive if $w < a - c + \delta$. This condition is always fulfilled at the equilibrium because, otherwise, the patent holder who defines w, would make no profit.

- With vertical integration. *With a duopoly equilibrium,* the quantity, price and profit of the firm s are:

$$q_s(w,\theta_s) = \frac{(2-\lambda)(a-c)-\lambda\delta+2\theta_s(\delta-w)}{b(4-\lambda^2)}$$
$$p_s(w,\theta_s) = c+\theta_s w + \frac{(2-\lambda)(a-c)-\lambda\delta+2\theta_s(\delta-w)}{4-\lambda^2}$$
$$\pi_s(w,\theta_s) = \frac{((2-\lambda)(a-c)-\lambda\delta+2\theta_s(\delta-w))^2}{b(4-\lambda^2)^2}$$

If $\theta_s = 0$, s sells positive quantities with a positive markup if $\delta < \delta_M$ (otherwise there is a monopoly equilibrium). If $\theta_s = 1$, s sells positive quantities with a positive markup if $w < (a-c+\delta)(2-\lambda)/2$. This condition is always fulfilled at the equilibrium.

The quantity, price and profit of the firm v are:

$$q_v(w,\theta_s) = \frac{(2-\lambda)(a-c)+2\delta-\lambda\theta_s(\delta-w)}{b(4-\lambda^2)}$$
$$p_v(w,\theta_s) = c + \frac{(2-\lambda)(a-c)+2\delta-\lambda\theta_s(\delta-w)}{4-\lambda^2}$$
$$\pi_v(w,\theta_s) = q_v(w,\theta_s)(p_v(w,\theta_s)-c) + q_s(w,\theta_s)\cdot w$$

There is no simple expression of the profit of the firm v.

With a monopoly equilibrium, quantities, prices and profits are:

$$q_v(0,0) = \frac{a-c+\delta}{2b} \qquad q_s(0,0) = 0$$
$$p_v(0,0) = c + \frac{a-c+\delta}{2} \qquad p_s(0,0) = c$$
$$\pi_v(0,0) = \frac{(a-c+\delta)^2}{4b} \qquad \pi_s(0,0) = 0$$

This equilibrium appears if $\theta_s = 0$ and $\delta \geq \delta_M$.

B Effect of vertical integration on the licensee's profit with a royalty

We show here that the profit of the licensee with a royalty (and no access restriction) is lower with vertical integration. Three cases need to be distinguished:

1) If $\delta > \delta_{NI}$, then $w_I^* = \tilde{w}_I$ and $w_{NI}^* = \tilde{w}_{NI}$. After compilation we have:

$$\pi_i(\tilde{w}_{NI},1,1) - \pi_s(\tilde{w}_I,1) = \frac{\lambda(64-32\lambda-7\lambda^2)}{4b(2+\lambda)^2(8-3\lambda^2)^2}\cdot(a-c+\delta)^2 > 0$$

2) If $\delta_I < \delta < \delta_{NI}$, then $w_I^* = \tilde{w}_I$ and $w_{NI}^* = \hat{w}_{NI}$. We have $\pi_i(\hat{w}_{NI},1,1) > \pi_i(\tilde{w}_{NI},1,1)$ because $\hat{w}_{NI} < \tilde{w}_{NI}$ and because profit of the licensee is decreasing in w. Moreover we showed that $\pi_i(\tilde{w}_{NI},1,1) > \pi_s(\tilde{w}_I,1)$ in the previous paragraph. Consequently, we have $\pi_i(\hat{w}_{NI},1,1) > \pi_s(\tilde{w}_I,1)$.

3) If $\delta < \delta_I$, then $w_I^* = \hat{w}_I$ and $w_{NI}^* = \hat{w}_{NI}$. After compilation we have:

$$\pi_i(\hat{w}_{NI},1,1) - \pi_s(\hat{w}_I,1) = \frac{\delta\lambda(2(2-\lambda)(a-c)-\lambda\delta)}{b(4-\lambda^2)}$$

This difference is always positive when $\delta < \delta_I$.

C Interest of the patent holder for vertical integration with a royalty

With a royalty license, we have seen that the patent holder have no interest for partial or complete access restriction. Three cases need to be considered as in the previous section.

1) If $\delta < \delta_I < \delta_{NI}$, then $w_I^* = \hat{w}_I$ and $w_{NI}^* = \hat{w}_{NI}$. Vertical integration increases the profit of the patent holder:

$$(\Pi_I(\hat{w}_I,1) - \pi_s(\hat{w}_I,1)) - (\Pi_{NI}(\hat{w}_{NI},1,1) - \pi_i(\hat{w}_{NI},1,1)) = \frac{\delta(\delta(4-4\lambda+\lambda^3)+\lambda^2(2-\lambda)(a-c))}{b(4-\lambda^2)^2} > 0$$

2) If $\delta_I < \delta < \delta_{NI}$, then $w_I^* = \tilde{w}_I$ and $w_{NI}^* = \hat{w}_{NI}$. The patent holder prefers to integrate vertically because of the the following inequalities:

$$\Pi_I(\tilde{w}_I,1) - \pi_s(\tilde{w}_I,1) > \Pi_I(\hat{w}_I,1) - \pi_s(\hat{w}_I,1) > \Pi_{NI}(\hat{w}_{NI},1,1) - \pi_i(\hat{w}_{NI},1,1)$$

The first part of the inequality comes from the fact that the profit of v is greater when the optimal royalty level is unconstrained rather than constrained. The second part of the inequality was established in the previous paragraph for any value of δ.

3) If $\delta_I < \delta_{NI} < \delta$, then $w_I^* = \tilde{w}_I$ and $w_{NI}^* = \tilde{w}_{NI}$. The patent holder prefers to vertically integrate because it leads to an increase of the industry profit and a decrease of the profit let to the licensee. The decrease of the licensee profit comes from the lemma 2. The increase of the industry profit can be shown has follow:

$$\Pi_{NI}(\tilde{w}_{NI},1,1) < \Pi_{NI}(\tilde{w}_I,1,1) < \Pi_I(\tilde{w}_I,1)$$

The first inequality comes from the fact $\tilde{w}_I < \tilde{w}_{NI}$ and that the industry profit is concave in w and decreasing when the optimal royalty level is unconstrained because, when $w = \tilde{w}_{NI}$, we have:

$$\left.\frac{\partial(\Pi_{NI}(w,1,1) - 2\pi_i(w,1,1))}{\partial w}\right|_{w=\tilde{w}_{NI}} = 0 \quad \Leftrightarrow \quad \left.\frac{\partial \Pi_{NI}(w,1,1)}{\partial w}\right|_{w=\tilde{w}_{NI}} = \frac{\partial 2\pi_i(w,1,1))}{\partial w} < 0$$

The second inequality comes from, first, that the industry profit with no access restriction is greater with vertical integration if the royalty is high enough ($w_{NI/I}$):

$$\Pi_I(w,1) > \Pi_{NI}(w,1,1) \quad \Leftrightarrow \quad w > w_{NI/I} \quad \text{with:} \quad w_{NI/I} = (a-c+\delta)\frac{\lambda(2-\lambda)^2}{4-3\lambda^2+2\lambda^3}$$

... and, second, that $\tilde{w}_I > w_{NI/I}$:

$$\tilde{w}_I - w_{NI/I} = \frac{(2-\lambda)(16-24\lambda+14\lambda^3+\lambda^4-2\lambda^5)}{2(8-3\lambda^2)(4-3\lambda^2+2\lambda^3)} \cdot (a-c+\delta) > 0$$

D Definition and properties of the fixed fee with no vertical integration

Since $w = 0$, the monopoly equilibrium appears if only one firm incorporates the innovation ($\theta_i \neq \theta_j$) and if the innovation is drastic ($\delta \geq \delta_M$).

- Definition of F_2. Remind that $F_2 = \pi_i(0,1,1) - \pi_i(0,0,1)$ and that the net profit of the licensee is $\pi_i(0,0,1)$. With a non-drastic innovation, the patent holder cannot extract all the profit of the licensees ($\pi_i(0,0,1) > 0$). Conversely, with a drastic innovation, the patent holder extracts all the profit of the licensees ($\pi_i(0,0,1) = 0$). The detailed expression of F_2 is:

$$F_2 = \begin{cases} \dfrac{4\delta((2-\lambda)(a-c)+\delta(1-\lambda))}{b(4-\lambda^2)^2} & \text{if } \delta < \delta_M \\ \dfrac{(a-c+\delta)^2}{b(2+\lambda)^2} & \text{otherwise} \end{cases} \tag{2}$$

- Definition of F_1. Remind that $F_1 = \pi_i(0,1,0) - \pi_i(0,0,0)$ and that the net profit of the licensee is $\pi_i(0,0,0)$. Whatever the type of innovation, the patent holder cannot extract all the profit of the licensees. ($\pi_i(0,0,0) > 0$). $\pi_i(0,1,0)$ corresponds to the duopoly profit if δ is small enough ($\delta < \delta_M$), and to the monopoly profit otherwise. The detailed expression of F_1 is:

$$F_1 = \begin{cases} \dfrac{4\delta((2-\lambda)(a-c)+\delta)}{b(4-\lambda^2)^2} & \text{if } \delta < \delta_M \\ \dfrac{(\lambda(a-c)+(2+\lambda)\delta)((4+\lambda)(a-c)+(2+\lambda)\delta)}{4b(2+\lambda)^2} & \text{otherwise} \end{cases} \tag{3}$$

- Comparison of F_2 and F_1. With a non-drastic ($\delta < \delta_M$) innovation we have:

$$F_1 - F_2 = \frac{4\lambda\delta^2}{b(4-\lambda^2)^2} > 0$$

With a drastic innovation, we have:

$$F_1 - F_2 = \frac{\lambda(4+\lambda)}{4(2+\lambda)^2}\delta^2 + \frac{\lambda(4+\lambda)}{2(2+\lambda)^2} \cdot (a-c)\delta - \frac{4-4\lambda-\lambda^2}{4(2+\lambda)^2} \cdot (a-c)^2$$

This expression is positive because it is convex in δ and the two roots are lower than δ_M.

E Conditions leading to access restriction with a fixed fee and no vertical integration

- With a non drastic innovation ($\delta < \delta_M$), access restriction leads to a duopoly equilibrium.

The variation of the patent holder profit when he chooses to restrict the access is:

$$\begin{aligned}[\Pi_{NI}(0,1,0) - \pi_i(0,0,0) - \pi_i(0,0,1)] - [\Pi_{NI}(0,1,1) - 2\pi_i(0,0,1)] \\ = -\frac{4(1-2\lambda)}{b(4-\lambda^2)^2}\delta^2 - \frac{4}{b(2-\lambda)(2+\lambda)^2}(a-c)\delta\end{aligned}$$

Two sub-cases have to be considered. If $\lambda \leq 0.5$, the variation is negative because it is concave in δ and the two roots are negative. If $\lambda > 0.5$, the variation is also negative because it is convex in δ, the lowest root is equal to 0 and the highest root is greater that δ_M. In summary, the patent holder does not prefer to restrict the access in this case.

- With a drastic innovation ($\delta \geq \delta_M$), access restriction leads to a monopoly equilibrium: the downstream firm which is foreclosed earns no profit ($\pi_i(0,0,1) = 0$). Conversely, there is still a duopoly equilibrium with no access restriction.

The variation of the patent holder profit when he chooses to restrict the access is:

$$\begin{aligned}[\Pi_{NI}(0,1,0) - \pi_i(0,0,0)] - [\Pi_{NI}(0,1,1)] \\ = \frac{-4+4\lambda+\lambda^2}{4b(2+\lambda)^2} \cdot \delta^2 + \frac{-4+4\lambda+\lambda^2}{2b(2+\lambda)^2} \cdot (a-c)\delta + \frac{-8+4\lambda+\lambda^2}{4b(2+\lambda)^2} \cdot (a-c)^2\end{aligned}$$

If $\lambda < 0.828$ this expression is negative because it is concave in δ and it has no root. If $\lambda > 0.828$ this expression is convex in δ, its smallest root is negative and its highest root is:

$$\delta_{NI}^F = \left(-1 + \frac{2}{\sqrt{-4+4\lambda+\lambda^2}}\right)(a-c)$$

Note that $\delta_{NI}^F > \delta_M$ and negative otherwise. Hence, the expression is positive if $\delta > \delta_{NI}^F$.

In summary, the patent holder prefers to restrict the access $\lambda > 0.828$ and if $\delta > \delta_{NI}^F$.

F Comparision of the licensee profit with royalty or fixed fee

This section provides the proof that, with no access restriction, the net profit of the licensee is lower with fixed fee compared to a royalty. This result holds both with and without vertical integration.

With vertical integration and a royalty, the licensee accepts the contract if the royalty is lower than a maximum level ($\hat{w}_I$ such that $\pi_s(\hat{w}_I, 1) = \pi_s(0, 0)$). Moreover, knowing that $w_I^* \leq \hat{w}_I$ and that the profit of the licensee is decreasing in w, we have:

$$\pi_s(w_I^*, 1) \geq \pi_s(\hat{w}_I, 1) = \pi_s(0, 0)$$

With no vertical integration, the net profit of the licensee with a fixed fee is lower compared to the case where nobody accepts the license ($\pi_i(0, 0, 1) \leq \pi_i(0, 0, 0)$). We have then:

$$\pi_i(w_{NI}^*, 1, 1) \geq \pi_i(\hat{w}_{NI}, 1, 1) = \pi_i(0, 0, 0) \geq \pi_i(0, 0, 1)$$

G Interest of vertical integration with a fixed fee or a royalty

- Interest for the patent holder

This section provides the proof that the patent holder always prefers to integrate vertically in the case where he prefers to apply a royalty (rather than a fixed fee) with vertical integration and the converse with no vertical integration. Two sub-cases have to be considered depending on whether the patent holder prefers to restrict the access restriction with no vertical integration (and a fixed fee).

Consider first the case where the patent holder does not restrict the access with no vertical integration. With vertical integration the patent holder prefers to apply a royalty, so we have:

$$\begin{aligned} & \Pi_I(w_I^*, 1) - \pi_s(w_I^*, 1) \geq \Pi_I(0, 1) - \pi_s(0, 0) \\ \Leftrightarrow \quad & \Pi_I(w_I^*, 1) - \pi_s(w_I^*, 1) \geq [\Pi_{NI}(0, 1, 1) - 2\pi_i(0, 0, 1)] + \pi_i(0, 0, 1) \end{aligned}$$

The equivalence is based on the fact that the profit with a fixed fee are equivalent with or without vertical integration (cf. lemma 4). The term on the right hand side of the second inequality is the joint profit of the patent holder (in squared brackets) and the licensee with fixed fee, no vertical integration, and no access restriction. Hence the patent holder prefers to integrate vertically.

The same type of equivalence can be established when the patent holder prefers to restrict the access with no vertical integration. There is also an interest for vertical integration because we have:

$$\begin{aligned} & \Pi_I(w_I^*, 1) - \pi_s(w_I^*, 1) \geq \Pi_I(0, 0) - \pi_s(0, 0) \\ \Leftrightarrow \quad & \Pi_I(w_I^*, 1) - \pi_s(w_I^*, 1) \geq [\Pi_{NI}(0, 1, 0) - \pi_i(0, 0, 0) - \pi_i(0, 0, 1)] + \pi_i(0, 0, 0) \end{aligned}$$

- Welfare impact of vertical integration

With no vertical integration, the patent holder always prefers a fixed fee to a royalty, but he can either or not prefers to restrict the access. With vertical integration, the patent holder can either prefers a fixed fee to a royalty or the converse, but he has no interest for access restriction.

Four potential potential cases have to be considered. Using the proposition 2, we can observe that it is not possible to have a case where the patent holders prefers a fixed fee and access restriction with no vertical integration, and a fixed fee plus access restriction with vertical integration. Hence, only three cases have to be considered.

1) If the patent holders prefers a fixed fee and no access restriction both with or without vertical integration, then the stage 3 equilibrium are identical both with or without vertical integration. The patent holder has no interest for vertical integration, and the social surplus is identical with or without vertical integration.

2) If the patent holders prefers a fixed fee and no access restriction with no vertical integration and a royalty with vertical integration, then he prefers to integrate vertically. The two products incorporate the innovation, but the prices are higher with vertical integration because of the royalty. Hence vertical integration leads to a decrease of the social surplus.

3) If the patent holders prefers a fixed fee and access restriction with no vertical integration, then he prefers a royalty to a fixed fee with vertical integration. With no vertical integration, the production cost is at the minimum level but the innovation is incorporated in only one product. With vertical integration, the innovation is incorporated in the two products but the production cost of the licensee is higher than the marginal cost because of the royalty. The compilation of the social surplus is necessary to analyse the welfare effect of vertical integration. Social surplus is defined as follow:

$$W(q_i, q_j, \theta_i, \theta_j) = q_i(a + \theta_i\delta - c) + q_j(a + \theta_j\delta - c) - \frac{b}{2} \cdot (q_i^2 + 2\lambda q_i q_j + q_j^2)$$

Vertical integration leads to an increase of the social surplus because:

$$W(q_v(w_I^*, 1), q_s(w_I^*, 1), 1, 1) - W(q_i(0, 1, 0), q_i(0, 0, 1), 1, 0) = \frac{(2-\lambda)(1-\lambda)(14-3\lambda-6\lambda^2)}{2(8-3\lambda^2)^2} \cdot \frac{(a+\delta-c)^2}{b} > 0$$

Note that the patent holder prefers to restrict the access with a fixed fee only if the innovation is radical ($\delta > \delta_M$), so that $q_i(0, 0, 1) = 0$.

In conclusion, if the patent holder prefers to integrate vertically, it leads either to an increase or a decrease of the social surplus depending on the access restriction he would decide with no vertical integration. Vertical integration leads to an increase (resp. decrease) of the social surplus if the patent holder would decide to restrict the access (resp. not to restrict the access) with no vertical integration. Moreover, there is no loss of surplus when the patent holder prefers not to integrate vertically.