Patent Scope and Technology Choice*

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Abstract

The purpose of this paper is to analyze the effect of an increase in patent scope on investments in R&D and on the rate of innovation. Patent scope affects incentives for innovation via the research strategies firms choose; a broad scope on the patent for the state-of-the-art technology can induce entrant firms to choose to do research on alternative technologies to avoid patent infringement. If the alternative technologies have a lower probability of success, this reduces incentives for investment in R&D by entrant firms and the probability that they innovate. On the other hand, the allocation of total R&D across projects is improved, since there is less wasteful duplication of R&D investments. This paper presents a model where the trade-off induced by patent scope can be analyzed. The model predicts that an increase in patent scope can increase the probability of innovation, and consequently the negative effects of R&D duplication can be large enough to warrant a broad patent scope. This holds if the incumbent's increase in profits from innovating is large, and the patented technology has a small advantage relative to alternative technologies. Otherwise, the probability of innovation decreases. However, when the model is extended to allow for Stackelberg competition or license agreements, the benefit of a broad patent scope to a large extent disappears. Hence, the effects of an increase in patent scope depend on innovation and industry characteristics and unless several conditions are met, an increase in patent scope reduces innovation.

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1 Introduction

According to several economists, it is widely perceived that the scope of intellectual property rights in the US has increased over the last two decades (See for example Jaffe 2000 and Gallini 2002). They point to two factors that suggest this is the case. Firstly, patent holders have been awarded greater power in infringement lawsuits by a broadening of the interpretation of patent claims. Secondly, patent protection has been extended to cover new areas, notably software, business methods and biotechnology, where a large number of patents with broad scope have been granted. The purpose of this paper is to analyze how an increase in patent scope affects subsequent investments in R&D and the rate of innovation.

Suppose that there are several research strategies firms can pursue in order to find the next generation product in a market. Either they do R&D using the patented state-of-the-art technology to make an incremental improvement on that technology, or they choose alternative R&D strategies. The alternative strategies entail using different technologies than the current state-of-the-art. If the state-of-the-art technology is covered by a patent which is broad in scope, that may induce firms other than the patent holder to pursue an alternative research strategy; use a different technology to avoid the risk of patent infringement.[1] Lerner (1995) finds that firms with high litigation costs tend to avoid research areas that are occupied by other firms, particularly when these firms have low litigation costs. Walsh, Arora and Cohen (2003) analyze the effect of patents on R&D in the pharmaceutical industry. They find that firms tend to direct R&D investment to research areas less covered by patents.

It is probable that pursuing an alternative research strategy is more costly or involves more uncertainty than pursuing a strategy which makes incremental improvements of the technology currently in use. A broad patent scope on the state-of-the-art technology therefore reduces the incentives for research by entrant firms, and their innovation rates. However, research efforts may be better allocated across potential projects. If many firms were to do R&D in order to develop the same technology there may be wasteful duplication of research investment. Firms may for example build parallel labs and carry out identical experiments or build identical prototypes, which from a welfare point of view is a waste of R&D resources. If they do R&D using different technologies, they are less likely to carry out identical experiments, and there is less wasteful duplication. Direct evidence of duplication of R&D are given by simultaneous innovation, which is common in science. Two examples discussed by Chatterjee and Evans (2004) are the parallel inventions of the first electronic mini-calculator by Casio and Texas Instruments in 1972, and the parallel discovery of the process for synthesis of leukotrienes by two competing research teams

[1] According to patent law, an invention which builds on a patented invention infringes that patent, even if it is patentable in its own right.

in 1979. Duplication of research effort which does not directly result in innovations is certainly more common. Domeij (2000) argues that patents that are broad in scope have resulted in diversification of research among pharmaceutical companies, as competitors to the patent holder strive to find pharmaceuticals based on other types of molecules than the one patented, which do not infringe on the patent.

In sum, an increase in patent scope has several effects on the investments in R&D and on the rate of innovation in the economy. It reduces entrant firms' incentives for research and their innovation rates. At the same time, an increase in patent scope decreases duplication of research effort, and directs research towards potentially fruitful new technologies and methods. The question is whether an improvement in allocation across projects can offset any decrease in individual firms' innovation rates, and if so, under what conditions.

In this paper, I construct a model to analyze this trade-off caused by patent scope. For simplicity, I use a duopoly model with an incumbent firm and an entrant firm. The incumbent owns a patent connected to the state-of-the-art technology, and is producing the corresponding product. There are two possible research strategies to follow: the first one is to build on the state-of-the-art technology, and the second to use an alternative technology, which is less promising. I consider two alternative scenarios for patent scope: one in which the patent scope on the state-of-the-art technology is narrow, and one in which the patent scope is broad. In the case of a narrow scope, both firms can choose to do research on the state-of-the-art technology. In the case of a broad scope, the entrant has to choose the alternative, less promising, technology in order to avoid infringing on the patent. I describe the possible equilibria that can arise under narrow and broad scope respectively, and compare the resulting R&D investments and probabilities of innovation.

The model suggests a new explanation for the empirical finding that incumbent firms have high innovation rates relative to entrants. In the standard R&D race models, the incumbent invests less than entrant firms due to the Arrow effect: the incumbent has a lower incentive to innovate since he by innovating to some extent replaces his current profits. In this model, when the incumbent firm holds a patent which is broad in scope, it gives him a monopoly on research that has the highest expected payoff. This effect can increase the incumbent's incentives for R&D sufficiently to outweigh the Arrow effect. Hence, the incumbent can be more likely to innovate than the entrant when he has an advantage originating from policy, namely the scope of the patent he owns.

The model predicts that if the incumbent firm has a high stand-alone incentive to innovate, i.e. the difference in his profits after versus before he innovates is large, or if the patented technology has a small advantage relative to other technologies, a broad patent scope gives a higher probability of innovation than a narrow scope. Hence, the negative effects of R&D duplication are under some conditions large enough to warrant a broad patent scope. Conversely, when the incumbent's stand-alone incentive to innovate is low, or the patented technology has a large advantage, a narrow scope gives a higher probability of innovation. If the incumbent is able to commit to an investment level or if license agreements can be made, the first result is partly reversed; in instances where previously the highest innovation probability was given by a broad scope, it is now obtained under a narrow scope. Hence, the benefit of a broad patent scope largely relies on the assumptions that the firms act simultaneously, and that there are no possibilities for license agreements. Consequently, the effects of an increase in patent scope depend on innovation and industry characteristics, and unless several conditions are met, an increase in patent scope reduces innovation.

The paper is organized as follows. The related literature is presented in Section 2, and an introduction to the determination of patent scope is given in Section 3. Section 4 describes the model. The equilibria of the model are characterized in Section 5. Section 6 describes the investments and the probabilities of innovation resulting from the narrow and broad patent scope respectively. Section 7 entails a comparison of the innovation probabilities, and describes the conditions under which each patent regime gives the highest innovation probability and the highest social surplus. Section 8 extends the model to allow for Stackelberg competition and licensing. Section 9 concludes.

2 Related literature

There is a large theoretical literature on the economic effects of intellectual property rights. An increasingly spreading view is that the current system of intellectual property rights in the US offers innovators too much protection of their innovations. Heller and Eisenberg (1998) argue that there is a "tragedy of the anticommons" in biomedical research as there are numerous patents to each separate building block for a new product. Acquiring the rights to use all of them is costly and potentially difficult, as the owners of the rights may have heterogeneous interests. Therefore, patenting can constitute an obstacle to future research. Similarly, Shapiro (2001) argues that in several industries, such as semiconductors and software, the current patent system is creating a patent thicket, an overlapping set of patents, which requires innovators of new technology to obtain licenses from multiple patent holders. The high transaction costs involved implies that stronger patent rights may stifle innovation. Bessen and Maskin (2002) show that when innovations are sequential stronger intellectual property rights protection may reduce innovation even in the case when there is only one patent holder. On the other hand, Green and Scotchmer (1995) also present a model of sequential innovation and find that a broad patent scope can be necessary to give the first innovator sufficient incentives to invest.

In the law and economics literature, Kitch (1977) argues that pioneering technologies should be granted patents with broad scope, since it will allow the innovator to coordinate further development of the technology by granting licenses and thereby wasteful duplication of effort is reduced. His view is challenged by Merges and Nelson (1990). Their argument is that uncertainty and high transaction costs of licensing reduce the effectiveness of coordination, and that broad patent scope can instead block technology development. Technical advance is likely to be faster when there is competition, as the patent holder then has higher incentives to develop his technology. Domeij (2000) discusses the trade-off between total investment in R&D and duplication of investments induced by patent scope in the context of the pharmaceutical industry. When the second generation product is a new indication, i.e. the same compound is used to treat other types of illnesses, Domeij argues that the patent holder has a high incentive to search for new innovations, since they are intended for new markets. In addition, the patent holder has a technological advantage over competitors in finding this type of innovations. Consequently, he concludes that a broad patent scope is preferable.

In the literature on firms' choices of research strategies, Dasgupta and Maskin (1987) find that competition encourages firms to choose research projects that are too similar from a welfare point of view. Chatterjee and Evans (2004) show that if the projects differ in other dimensions than the probability of leading to an innovation, firms may either choose projects that are too similar, or projects that are too different relative to what is socially optimal. Previous literature on duplication of effort in research and development includes for example Tandon (1983), Jones and Williams (2000) and Zeira (2003). They model identical firms and do not take into account the different incentives facing incumbent and entrant firms. Cabral and Polak (2004) present a duopoly model with an incumbent and an entrant. They investigate how an increase in consumer valuation of the incumbent firm's good, interpreted as an increase in its dominance, affects the amount of duplication of R&D by the two firms and the rate of innovation. Their conclusion is that increased dominance has a positive effect on innovation when intellectual property rights are strong. However, neither of the models has a mechanism by which entrant firms' technology choices affect the duplication of R&D.

This paper is also related to the literature concerned with why incumbent firms have high innovation rates relative to entrant firms. As shown by Reinganum (1983), when the innovation process is stochastic an incumbent firm invests less in R&D than an entrant, and is less likely to innovate. However, empirical evidence points to the opposite. For example, Blundell et al. (1999) find that within industries, firms with high market share innovate more. Several explanations for this observation have been proposed, most of them relying on a technological advantage for the incumbent. One example is Segerstrom and Zolnierek (1999) where the incumbent has lower costs of R&D than entrant firms. Another is Etro (2004), where the explanation is a first mover advantage for the incumbent in combination with free entry.

3 Determination of patent scope

The scope of a patent is central to this analysis. Therefore, I will start with a brief introduction to the determination of patent scope in patent law and practice, as described in Merges and Nelson (1990). A patent application consists of a specification of the innovation and a set of claims. The specification is written as an engineering article and describes the problem the innovator faced, and how it was solved. The claims define what the inventor considers to be the scope of the innovation, the "technological territory" in which he can sue other parties for infringement. The general rule is that a patent's claims should extend beyond the precise disclosure of the innovation in the specification. Otherwise, imitators could make minor changes to that example without infringing and the patent would have little value. The inventor naturally wants to make the claims as broad as possible, and the patent examiner must decide what scope is appropriate, which claims should be admitted and which should not.

In infringement cases the court first examines whether there is "literal infringement", namely the product literally falls within the boundaries of the patent claims. If not, the court also examines whether the product infringes under the doctrine of equivalents. The doctrine of equivalents says that a product is infringing if it does the work in substantially the same way and accomplish substantially the same result as the patented product. Consequently, patent scope is determined in two instances, by two separate authorities. Ex ante, if the patent holder has not sued any other party for infringement, the patent scope is defined by the claims as determined by the Patent Office. Ex post, in an infringement case, the patent scope is determined by the court, in its decision whether the patent has been infringed.

4 Model

The economy has two firms, an incumbent and an entrant. Both firms make investments in research and development in order to find the next innovation, which has private value V when patented. Both firms have quadratic investment costs. The incumbent firm holds a patent connected to the current state-of-the-art technology, and earns a profit from producing the corresponding product. The profit is expressed as a share of the value of the next innovation, αV, where $\alpha \in [0, 1]$. The entrant earns no current profits. Innovation is drastic; new innovations replace previous ones.

There are two possible research strategies for a firm to pursue. Strategy C is to build on the current state-of-the-art technology, technology C, and make an improved product. Strategy A is to use an alternative technology, technology A, for which there is no risk of patent infringement. In this context, a technology should be interpreted more broadly as using another material, algorithm, chemical compound etc., depending on the industry and the nature of the product. Irrespective of which technology is used in R&D, the private value of an innovation is V. There is no strong reason to believe that using different technologies to develop a certain product will generate innovations of exactly the same value. However, this simplification enables me to distinguish the effects of different patent regimes on innovation from effects of a higher value of an innovation.

Each technology has an exogenous probability γ_i, $i \in \{C, A\}$, of leading to the next innovation. The alternative technology has a lower probability of leading to the next innovation than the state-of-the-art: $\gamma_A < \gamma_C$. The difference between γ_C and γ_A reflects the relative advantage the state-of-the-art technology has. I assume that either technology C or A leads to a new innovation, but not both. This is a simplification of technology development, but is made for tractability. I will discuss the implications of the assumption further below. In addition, I normalize the sum of γ_A and γ_C to 1, since it reduces the number of model parameters. Hence, $\gamma_A = 1 - \gamma_C$. This does not affect the main results, since what is important for the mechanisms of the model is the ratio $\frac{\gamma_C}{\gamma_A}$.

In this paper, the R&D process is modeled as a one-shot game. This modelling choice is motivated by the fact that firm's R&D projects for development of new products often are close to irreversible. This is especially true in the biotechnology and pharmaceutical industries. As a consequence of this structure, the model has a positive probability that both firms innovate if they choose the same technology to do R&D on. It is necessary to specify the payoffs to both firms if this event occurs. Let the game be interpreted as a time period of for example five years, a period over which it is reasonable to assume that the R&D strategy cannot easily be changed. If both firms innovate during this time period, a patent will be awarded to the firm which innovated first. Suppose that innovations arrive with a hazard rate that depends on the R&D investment. Then, conditional on both innovating, the firms have the same probability of innovating at each point in time. Therefore, I assume that each firm has probability $\frac{1}{2}$ of obtaining the patent if both innovate. Further, I assume that the incumbent always chooses to invest in technology C irrespective of the entrant's technology choice. A justification for this assumption is that using a particular technology requires a fixed cost or an investment in human capital.[2] In the baseline model, both firms act simultaneously. In section 8, the model is extended to Stackelberg competition.

The patent regimes are modeled as follows. The patent scope on the state-of-the-art technology can be either narrow or broad. In the case of a narrow patent scope, the entrant can choose between the two technologies when investing in R&D, and he selects the technology which gives the highest expected

[2] The assumption rules out an equilibrium in which the incumbent would choose to abandon his patented technology and invest in an alternative technology that is ex ante less attractive, only in order to escape competition from the entrant.

payoff. In the case of a broad patent scope, the incumbent can use his patent to block the entrant's innovation if it is based on technology C. Therefore, the entrant automatically chooses technology A in order to avoid infringement. This assumption will be relaxed in Section 8, where the model is extended to allow for license agreements between the firms.

The possibility of duplication of R&D resources can be illustrated in terms of two urns, A and C, filled with marbles. Each urn corresponds to a technology. Suppose that a firm's investment in R&D can be described as drawing a number of marbles from one of the urns and then replacing them. Drawing one marble is equivalent to conducting one experiment. Each urn has its own set of marbles, and the number of marbles is n_i, $i \in \{C, A\}$. Only one marble corresponds to a successful experiment, i.e. an innovation, and this marble is denoted 1. With probability γ_C marble number 1 is in urn C. Firm j purchases t_j, $j \in \{I, E\}$ marbles from urn i and the probability that it innovates, conditional on having chosen the right urn, is $\frac{t_j}{n_i}$. Firm j can increase this probability by buying more marbles at a cost of the per marble price. The draws of different firms are independent events. Suppose first that the incumbent and the entrant both choose urn C. The incumbent draws t_I marbles and replaces them in the urn, which gives him an innovation probability $\frac{t_I}{n_C}$. Then, the entrant draws t_E marbles, resulting in an innovation probability $\frac{t_E}{n_C}$. It is possible that both firms draw the same marble, that is, conduct the same experiment. This is a duplication of R&D resources from the point of view of society. No individual firm draws the same marble twice, and there are no duplicate experiments on the firm level. A social planner is however interested in the probability of any of the firms drawing marble number 1. For two events A and B, we can write the probability of at least one event occurring as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The two events are independent, hence $P(A \cap B) = P(A)P(B)$. In our example, the probability of at least one innovation is:

$$P(no\ 1\ at\ least\ once) = \gamma_C \left(\frac{t_I}{n_C} + \frac{t_E}{n_C} - \frac{t_I}{n_C}\frac{t_E}{n_C} \right)$$

The product $\frac{t_I}{n_C}\frac{t_E}{n_C}$ represents a waste of resources due to duplication.

Now, suppose instead that the incumbent draws marbles from urn C and the entrant draws marbles from urn A. The incumbent and entrant have probabilities $\frac{t_I}{n_C}$ and $\frac{t_E}{n_A}$ respectively, of drawing marble number 1, conditional on choosing the right urn. The probability that both firms draw the same marble is zero. Hence, the probability of at least one innovation is:

$$P(no\ 1\ at\ least\ once) = \gamma_C \frac{t_I}{n_C} + (1 - \gamma_C)\frac{t_E}{n_A}$$

There is no waste of resources due to duplication. Next, I turn to the characterization of the equilibrium investments in R&D.

5 Equilibrium investments

In this model, the incumbent always invests in his patented technology when he does R&D. He invests the amount of resources, p_I, which maximizes his expected payoff Π_I, where subscript I denotes incumbent. The entrant, on the other hand, chooses both which technology to invest in, denoted i, and the level of investment, p_E, which maximizes his expected payoff Π_E, where subscript E denotes entrant. Each firm's investment translates directly into its probability of innovating. The timing of the game is as follows: First, the entrant chooses which technology to invest in. Second, given the entrant's technology choice, both firms simultaneously decide how much to invest. An equilibrium is a triplet $\{i^*, p_I^*, p_E^*\}$, $i \in \{C, A\}$ and $p_I, p_E \in [0,1]$ such that $i^*, p_E^* = \arg\max_{i,p_E} \Pi_E(i, p_I^*, p_E)$ and $p_I^* = \arg\max_{p_I} \Pi_I(i^*, p_I, p_E^*)$. I divide the equilibria into two types given the entrant's choice of i:

- If the entrant chooses C, the equilibrium is of type C
- If the entrant chooses A, the equilibrium is of type A

First, the investments in equilibrium of type C are characterized, and then investments in equilibrium of type A. In order to interpret the firms' investments as probabilities of innovation, each investment must be bounded above by 1. I focus on the case when the optimal investment levels by both firms are interior solutions. In the baseline model, this is achieved by setting both V and the marginal cost of investment equal to 1. The effects of varying V will be analyzed in Section 7.

5.1 Equilibrium of type C

The expected payoff to the incumbent when both firms choose technology C is

$$\begin{aligned}\Pi_I(C, p_I, p_E) &= \alpha V + \gamma_C p_I(1-p_E)(V-\alpha V) + \gamma_C p_E(1-p_I)(0-\alpha V) \\ &\quad + \gamma_C p_E p_I\left(\frac{1}{2}(V-\alpha V) + \frac{1}{2}(0-\alpha V)\right) - \frac{(p_I)^2}{2} \qquad (1)\end{aligned}$$

where the subscript I denotes incumbent. With probability $\gamma_C p_I(1-p_E)$ the incumbent innovates whereas the entrant does not. The gain is $V(1-\alpha)$, the value of the innovation net of current profit, since the new product replaces the old one. Following Katz and Shapiro (1987), I refer to $V(1-\alpha)$ as the incumbent's stand-alone incentive to innovate, i.e. the difference in profit after versus before he innovates if he believes his rival will not innovate. With probability $\gamma_C p_E(1-p_I)$ the entrant innovates but not the incumbent, and the latter loses his current profits. With probability $\gamma_C p_E p_I$ both firms innovate, in which case the incumbent has probability $\frac{1}{2}$ of obtaining the patent. The variable cost of R&D is $\frac{(p_I)^2}{2}$. The first order condition yields

$$p_I = \gamma_C V\left(1 - \alpha - p_E\left(\frac{1}{2} - \alpha\right)\right)$$

The incumbent's investment p_I is increasing in γ_C and in $V(1-\alpha)$. It can be increasing or decreasing in p_E, depending on the value of α. There are two opposing forces at work: A higher investment by the entrant lowers the probability that the incumbent wins the patent, given his own investment, which decreases his incentives to invest in order to win. At the same time, a higher investment by the entrant increases the probability that the entrant wins. This increases the incumbent's returns to investing in order not to lose current profit and to increase the probability that both innovate, which increases his expected payoff by $\frac{V}{2}$. This effect increases the incumbent's incentive to invest. When $\alpha > \frac{1}{2}$, current profits are high relative to the value of innovation and expected payoff from winning is low. The latter effect dominates. When, $\alpha < \frac{1}{2}$, current profits are low relative to the value of innovation, and the first effect dominates. The cutoff point is at $\alpha = \frac{1}{2}$, which follows from the assumption that if both firms innovate, each firm has probability $\frac{1}{2}$ of obtaining the patent.

The expected payoff to the entrant when both firms choose technology C is

$$\Pi_E\left(C, p_I, p_E\right) = \gamma_C p_E (1-p_I) V + \gamma_C p_E p_I \frac{1}{2} V - \frac{\left(p_E\right)^2}{2} \tag{2}$$

With probability $\gamma_C p_E(1-p_I)$ the entrant wins V. With probability $\gamma_C p_E p_I$ both firms innovate, in which case the entrant gets V with probability $\frac{1}{2}$. The first order condition yields

$$p_E = \gamma_C V \left(1 - \frac{p_I}{2}\right) \tag{3}$$

The entrant's investment is decreasing in the incumbent's, for all parameter values. Solving for Nash equilibrium investment levels, given $V = 1$, yields the following investment by the incumbent and the entrant, respectively.

$$p_I^C(\alpha, \gamma_C) = \frac{2\gamma_C\left(2(1-\alpha) + 2\alpha\gamma_C - \gamma_C\right)}{\left(4 + 2\alpha\gamma_C^2 - \gamma_C^2\right)} \tag{4}$$

$$p_E^C(\alpha, \gamma_C) = \frac{2\gamma_C\left(2 + \alpha\gamma_C - \gamma_C\right)}{\left(4 + 2\alpha\gamma_C^2 - \gamma_C^2\right)} \tag{5}$$

where the superscript C indicates that the equilibrium is of type C. The incumbent's equilibrium investment $p_I^C(\alpha, \gamma_C)$ is increasing in γ_C and decreasing in α. The entrant's equilibrium investment $p_E^C(\alpha, \gamma_C)$ is increasing in γ_C and α. An increase in γ_C implies a higher probability that technology C leads to the next innovation, which increases both firms' investments. An increase in α decreases the incumbent's stand-alone incentive to innovate, $V(1-\alpha)$, which reduces his investment. The entrant responds to this reduction by increasing his investment.

As long as $\alpha > 0$, $p_I^C(\alpha, \gamma_C) < p_E^C(\alpha, \gamma_C)$. The fact that the incumbent stands to lose current profit from innovating, while the entrant does not, implies that in equilibrium the incumbent invests less. This is the Arrow effect. When the two firms invest in the same technologies, innovation is characterized by leapfrogging, i.e. the incumbent is less likely than the entrant to be the next innovator.

5.2 Equilibrium of type A

The expected payoff to the incumbent when the entrant chooses technology A is

$$\Pi_I(A, p_I, p_E) = \alpha V + \gamma_C p_I(V - \alpha V) + (1-\gamma_C)p_E(0 - \alpha V) - \frac{(p_I)^2}{2}$$

The assumption that one of the technologies leads to innovation, but not both, implies that the incumbent's optimal investment is independent of that of the entrant's. Taking the first order condition, given $V = 1$, yields

$$p_I^A(\alpha, \gamma_C) = \gamma_C(1-\alpha)$$

where the superscript A indicates that the equilibrium is of type A. The expected payoff to the entrant when he chooses technology A is

$$\Pi_E(A, p_I, p_E) = (1-\gamma_C)p_E V - \frac{(p_E)^2}{2}$$

Taking the first order condition, given $V = 1$, yields

$$p_E^A(\gamma_C) = (1-\gamma_C)$$

As above, the entrant's optimal investment is independent of the investment by the incumbent.

In this equilibrium, the incumbent invests in a technology that is more likely to lead to the next innovation, which increases his incentives to invest relative to the entrant's. If this effect is sufficiently strong, it can dominate the Arrow effect. If the following condition holds

$$\gamma_C > \frac{1}{2-\alpha} \tag{6}$$

the incumbent is more likely to innovate than the entrant. The threshold value for γ_C defined by (6) is increasing in α and takes values in the interval $\left(\frac{1}{2}, 1\right)$. A lower stand-alone incentive for the incumbent to innovate requires a higher probability of success for technology C in order to offset it.

5.3 The entrant's choice of technology

Let us return to the entrant's decision of which technology to invest in. The entrant chooses the technology which gives the highest expected payoff, given the equilibrium investments described above. The condition for when choosing C has a higher expected payoff than choosing A is given below.

Proposition 1 *If (7) holds, then the Nash equilibrium is of type C.*

$$\alpha > \frac{4 - 8\gamma_C + \gamma_C^2 + \gamma_C^3}{2\gamma_C^3} = \bar{\alpha} \tag{7}$$

Proof. See Appendix ■

The higher is α, the lower will the incumbent's investment be, which increases the entrant's expected payoff from choosing C relative to A. The threshold in (7) is decreasing in γ_C, since a larger probability of success for technology C increases the entrant's relative expected payoff from choosing C. I assume that if indifferent, the entrant chooses technology A.

6 Patent scope

In this model, the scope of a patent can be either narrow or broad. Patent scope is defined such that if the scope of the patent connected to technology C is narrow, the entrant can choose between technology C and A and hence the possible types of equilibria are both C and A. If the patent scope is broad, the entrant has to choose technology A in order to avoid patent infringement, and the equilibrium is always of type A. Consequently, patent scope determines which strategies are available to the entrant.

I assume that the patent scope does not affect V, the private value of the innovation, or α, the incumbent's current profit relative to V. One may argue that a broad scope can increase the current profits accruing to the patent holder as it discourages development of substitutes during the patented product's life. This effect would reduce the incumbent's investment in R&D under a broad scope relative to a narrow. The assumption that α is independent of scope gives an upper bound to the incumbent's investment under a broad scope. One may also argue that a firm's expectation of patent scope affects the expected value of innovating. In a dynamic model, V would correspond to the present discounted value of future profits, and if future patents are expected to be broad in scope, that may translate into a higher V, given expectations of γ_C and α. Hence, expectations of patent scope can affect firms' investments independently of their technology choices, but to assess the magnitude of this effect a dynamic model is required. In this paper, I abstract from the potential effects of patent scope on innovation through current profit and expectations of future scope, and analyze the effect through technology choice alone. Nevertheless, as a robustness check I also allow V to take a higher, exogenously given value under a broad scope relative to a narrow, to assess the impact on the model's predictions. The result is reported in Section 7.

In the case of a narrow patent scope, the entrant will choose C if (7) holds. If not, patent scope is irrelevant for the entrant's technology choice, as he chooses technology A under a narrow patent scope as well as under a broad. Therefore, the comparison of investment and innovation probabilities under differing patent scope is meaningful only under the condition $\alpha > \bar{\alpha}$.

6.1 Narrow patent scope

I start with a characterization of the investments by the two firms and the aggregate innovation probability under a narrow patent scope. Suppose that $\alpha > \bar{\alpha}$ so that the equilibrium is C. In this type of equilibrium, the entrant is more likely to innovate than the incumbent, and there will be leapfrogging, as in the standard stochastic racing and endogenous growth models.

The aggregate innovation probability is defined as the probability of at least one firm innovating. When both the entrant and the incumbent invest in the same technology there is duplication of R&D investment, and in analogy with the example in Section 4 the innovation probability is:

$$i^N = \gamma_C \left[p_I^C(\alpha, \gamma_C) + p_E^C(\alpha, \gamma_C) - p_I^C(\alpha, \gamma_C)\, p_E^C(\alpha, \gamma_C) \right] \tag{8}$$

where the superscript N denotes narrow patent scope. The amount of duplication for a given total investment is highest when the two firms' investments are equal, and it decreases as investments become more asymmetric. It reflects the fact that no firm duplicates its own research, but the higher is the potential overlap in experiments with the other firm, the higher is the probability of duplication. The innovation probability i^N is increasing in γ_C. Inspection of $\frac{\partial i^N}{\partial \alpha}$ shows that i^N is increasing in α if

$$\alpha > \frac{(\gamma_C - 2)^2}{2\gamma_C^2}$$

and decreasing otherwise. An increase in α decreases the incumbent's investment, and increases the entrant's. The net effect is a decrease in total investment, but also a decrease in duplication as the investments become more unequal. When α is sufficiently high, the reduction in total investment is offset by the decrease in duplication.

6.2 Broad patent scope

Now, I turn to a characterization of investments and the innovation probability under a broad patent scope. A broad patent scope implies that firms are in an equilibrium of type A. If γ_C is sufficiently high so that (6) holds, the incumbent is more likely to innovate than the entrant. The fact that the entrant has a monopoly on the more promising technology, given by the broad scope of the patent, provides him with an additional incentive to invest. If the patented technology has a sufficiently large advantage relative to the alternative, the incumbent becomes more likely to innovate than the entrant. Under a narrow scope, in contrast, the entrant is always more likely to innovate, irrespective of the value of γ_C. As described above, Etro (2004) explains the empirical pattern of innovation by incumbents with a first mover advantage for the incumbent, and Segerstrom and Zolnierek (1999) among others, with a technological advantage. This paper suggests an additional source of advantage for the incumbent resulting from policy, namely the scope of his patent.

The innovation probability under a broad patent scope is

$$i^B = \gamma_C p_I^A(\alpha, \gamma_C) + (1 - \gamma_C) p_E^A(\gamma_C) \tag{9}$$

where the superscript B denotes broad patent scope. Note that there is no duplication of R&D. It follows that i^B is decreasing in α. It is increasing in γ_C if (6) holds.

7 Effects of patent scope on innovation

The previous Section characterized the economy's innovation probability under the two patent regimes; a narrow and a broad scope, respectively. The effects of differing patent scope on innovation can be analyzed by comparing the ratio of the resulting innovation probabilities

$$\frac{i^N}{i^B} = \frac{\gamma_C \left[p_I^C(\alpha, \gamma_C) + p_E^C(\alpha, \gamma_C) - p_I^C(\alpha, \gamma_C) p_E^C(\alpha, \gamma_C) \right]}{\gamma_C p_I^A(\alpha, \gamma_C) + (1 - \gamma_C) p_E^A(\gamma_C)} \tag{10}$$

First, I describe how $\frac{i^N}{i^B}$ varies with the two key parameters in the model: α and γ_C.

Proposition 2 $\frac{i^N}{i^B}$ *is increasing in* α*.*

Proof. See Appendix ■

If the incumbent has a low stand-alone incentive to innovate, α is high, then the benefit of introducing competition in R&D on the current technology is large. The entrant invests more in case he gets access to this technology. In addition, the small investment by the incumbent relative to that of the entrant implies a low amount of duplication.

Proposition 3 *For all* $\alpha \in (0, 1)$ $\arg\max_{\gamma_C} \left(\frac{i^N}{i^B} \right) < 1$.

Proof. See Appendix ■

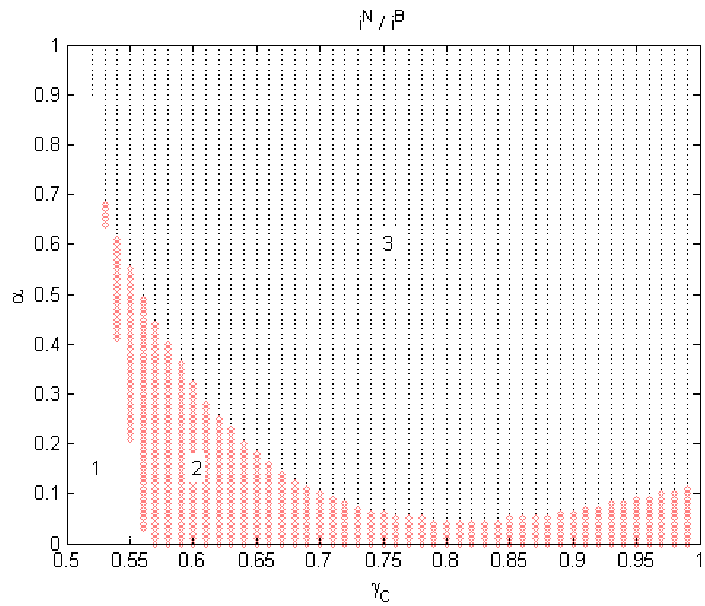
The ratio $\frac{i^N}{i^B}$ takes its highest value for $\gamma_C < 1$. The numerical solution shows that $\frac{i^N}{i^B}$ has an inverted U-shape over γ_C for all values of $\alpha \in (0, 1)$. One might have thought that the largest gain from a narrow scope would be obtained when the patented technology leads to the next innovation with probability 1, that is when there are no expected gains from doing research on an alternative technology. The intuition for this result is that for γ_C close to 1, an increase in γ_C increases total investment under both a narrow and a broad scope, but under a narrow scope there is a high degree of duplication. The duplication effect implies that $\frac{i^N}{i^B}$ decreases.

Now, I return to the assumption that both technologies cannot simultaneously lead to an innovation. This assumption is made for tractability, rather than as a description of reality. Relaxing the assumption will have the following implication for the results. Given a positive probability that both firms find an innovation when they invest in different technologies, there is now strategic interaction between the two firms in equilibrium of type A, which reduces the level of total investment in that equilibrium. Suppose both firms find an innovation. Each firm can obtain a patent, and if they can collude, each firm will earn $\frac{V}{2}$ from selling its innovation. If not, the expected gains from innovation are lower, which further reduces the level of investment in equilibrium of type A. Consequently, the assumption does not affect the the main results of the model, but introduces a level effect on the investments under a broad scope. Hence, it gives an upper bound on the benefit of a broad scope compared to a narrow scope.

7.1 Does a broad scope give a higher probability of innovation?

In order to assess the effects on innovation of an increase in patent scope, it is instructive to return to the trade-off between total investment and allocation of investment. A narrow patent scope allows both firms to do research on the most promising technology, but gives rise to duplication of R&D. This effect decreases the numerator of $\frac{i^N}{i^B}$. A broad patent scope forces the entrant to do research that is ex ante less promising, and he has a lower probability of innovation, which decreases the denominator of $\frac{i^N}{i^B}$. In order to answer the question: Does a broader scope give a higher probability of innovation?, it remains to determine which effect dominates, and under what conditions. That is, when is $i^B > i^N$ and vice versa? I solve for the innovation probabilities for all values of $\gamma_C \in [0.5, 1]$ and $\alpha \in [0, 1]$ and the result is shown in Figure 1.

Figure 1



Area 1: patent scope is inconsequential.Area 2: $i^N < i^B$.Area 3: $i^N > i^B$.

In Figure 1, the area labelled 1 is the area in which the entrant chooses equilibrium A even under a narrow scope and the patent scope has no effect on the innovation probabilities. The area labelled 2 is the one in which the broad scope gives the highest probability of innovation, whereas area 3 is the one in which a narrow scope gives the highest probability of innovation. Figure 1 shows that a broad scope gives a higher innovation probability for low values of γ_C and α, that is when the patented technology has a small advantage relative to the alternative and the incumbent's stand-alone incentive to innovate is high. When technology C has a small advantage, the entrant if forced to do R&D on technology A, does not reduce his investment very much. Consequently, a broad patent scope gives a higher probability of innovation. When the incumbent has a high stand-alone incentive to innovate, the amount of duplication under a narrow scope is high and a broad patent scope gives a higher probability of innovation. As seen in the Figure, for a lion's share of the parameter space, a narrow patent scope gives a higher probability of innovation.

7.2 Example from the biotechnology industry

The model developed in this paper can be used to assess whether granting a broad scope on a patent in a given market had a positive or negative impact on innovation in that market. As an illustration, consider an example of a specific patent in biotechnology. As noted in the introduction, biotechnology is an industry where a number of patents with broad scope have been granted.

The patent is on "Linked breast and ovarian cancer susceptibility gene", US patent [#5,709,999], which is owned by the National Institutes of Health, the University of Utah and the firm Myriad Genetics. The patented innovation is a method for diagnosing breast cancer. The technology used is certain mutations in the gene BRCA1 which have shown to increase a woman's probability of developing breast cancer. If a firm wants to find a new generation diagnostic method in this market, it can do R&D on the patented technology, that is search for an improved diagnostic method which identifies these mutations in BRCA1. Another option is to do R&D on an alternative technology, that is find an improved diagnostic method which identifies new mutations in BRCA1, or mutations in a different gene. The scope of this patent is broad; it covers all diagnostic tests identifying these mutations in BRCA1, irrespective of how the tests are performed[3]. In order to avoid infringement, all firms other than Myriad itself must do R&D on an alternative technology.

In order to apply the model to this example, it is necessary to assign values to the parameters γ_C and α. Starting with γ_C, the patented technology's advantage, the following argument can be made. The alternative technology relies on using other mutations than the ones described in the patent. However, these other mutations must first be identified, connected to the development of this form of breast cancer and be shown to be reliable indicators. Therefore, it is probable that doing R&D on the mutations in BRCA1 covered by the patent has a much higher probability of success than doing R&D on alternative mutations. In the model, this corresponds to a high γ_C. Next, what is $1 - \alpha$, the incumbent's stand-alone incentive to innovate? According to Orsi and Coriat (2005), the incentive is low. Myriad has not made improvements on its own test, which uses a direct sequencing method. However, French researchers at the Institute Curie have argued that Myriad's test fails to detect 10-20 percent of mutations, and that it would be possible to develop tests with higher precision, relying on "combing" techniques. The fact that no improvements have been made, suggests a high value of α. For an industry characterized by a high γ_C and a high α, the model predicts that a narrow patent scope gives the highest probability of innovation. Hence, according to the model, assigning a broad scope rather than a narrow on the patent on "Linked breast and ovarian cancer susceptibility gene" has reduced the probability of innovation in breast cancer diagnostics for the US market.

7.3 Social surplus

The previous Section shows under what conditions a broad and a narrow patent scope, respectively, gives the highest probability of innovation. However, maximizing the probability of innovation is desirable only insofar it is also socially optimal. In addition to the duplication effect, a social planner must take two other effects of R&D into account when choosing patent scope. The first effect is the social value of innovation, which is typically considered to be larger

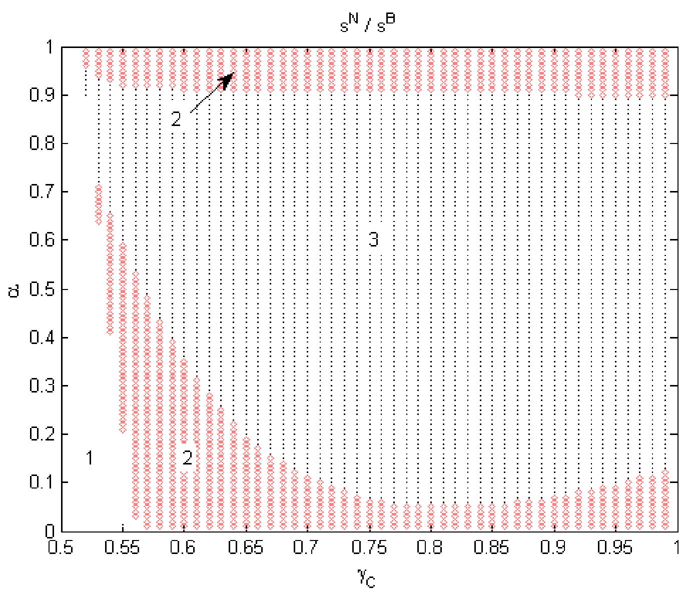
[3] See Nature, Vol. 418, July 2002

than the private value. One reason is that creation of new knowledge generates spillovers across sectors in the economy and across time. The second effect is the business stealing effect; entrant firms do not take into account the fact that as they innovate, the incumbent's profit is lost. In order to analyze which patent scope is socially optimal in this model, I define the social surplus as s^k where $k \in \{N, B\}$ denotes the patent scope. I assume that the private value of innovation is proportional to the social value. In addition, the social value of the new innovation is S and the social value of the current innovation is αS. The increase in social value from innovation is then $S(1-\alpha)$, when accounting for the business stealing effect, and it comes at a cost equal to the sum of the two firms' investment costs. The ratio of social surpluses is

$$\frac{s^N}{s^B} = \frac{i^N S(1-\alpha) - \frac{\left(p_I^C(\alpha,\gamma_C)\right)^2 + \left(p_E^C(\alpha,\gamma_C)\right)^2}{2}}{i^B S(1-\alpha) - \frac{\left(p_I^A(\alpha,\gamma_C)\right)^2 + \left(p_E^A(\gamma_C)\right)^2}{2}}$$

Suppose that the social value is 5 times larger than the private value of innovation, $S = 5V$. I solve numerically for the social surpluses to see when $\frac{s^N}{s^B} > 1$. The result is shown in Figure 2.

Figure 2



Area 1: patent scope is inconsequential.Area 2: $s^N < s^B$.Area 3: $s^N > s^B$.

In the comparison of social surpluses in Figure 2, it is notable that for most values of α, the patent scope which maximizes the probability of innovation is also the scope that is socially optimal. However, when α is close to 1, a broad scope, which gives the lowest probability of innovation in this parameter range, gives the highest surplus. The reason is that the innovation generates such a small increase in social value that it is optimal to restrict the investments in R&D. The level of α above which restricting investments is optimal depends on the ratio of social to private value of innovation, which in this example was set to 5. If the ratio is large enough, restricting investment will never be optimal. The tentative conclusion is that the socially optimal patent scope is that which maximizes the probability of innovation, except when the increase in social value from the innovation is small.

7.4 Effects of varying the private value of innovation

In the baseline model, I have set $V = 1$ in order to assure that equilibrium investments are bounded above by 1. This precludes any analysis of the effects of varying the private value of innovation. Now, I allow for corner solutions where $p_I, p_E = 1$, and analyze the effects of an increase in V. It is still assumed that both technologies give rise to innovations of equal value. The result is that an increase in the value of innovation has two effects. First, compared to Figure 1 it increases the area of parameter space for which patent scope is inconsequential. The intuition for this result is that an increase in V increases the incumbent's investment, which decreases the entrant's payoff in equilibrium of type C but not A. Second, it increases the area of parameter space for which a broad scope gives a higher innovation probability than a narrow scope. The reason is that an increase in V increases the investment by both firms, but under a narrow scope there is also an increase in the amount of duplication.

As a robustness check, I also allow V to take a higher, exogenously given value under a broad patent scope relative to a narrow. If firms expect the value of innovation to be higher under a broad scope, that increases incentives to invest under a broad scope relative to a narrow, and one may expect a decrease in $\frac{i^N}{i^B}$. Let $V^B = \theta V^N, \theta > 1$. First, the model is solved for $\theta = 1.5$; firms expect that obtaining a broad patent scope increases the value of innovation by 50 percent. The result is an increase in the area of parameter space for which a broad scope gives a higher innovation probability than a narrow scope, as compared to Figure 1. However, it is still the case that a broad patent scope gives the highest innovation probability for *less* than half of the total area of parameter space spanned by α and γ_C. The model is also solved for $\theta = 2$; firms expect the broad patent scope to double the value of innovation, which should be an upper bound on the differences in V caused by patent scope. Nevertheless, a broad patent scope gives the highest innovation probability only for *less* than two thirds of the total area of the parameter space.

8 Extensions of the model

Until now, it has been assumed that both firms simultaneously decide how much to invest. In addition, it has been assumed that the entrant cannot enter into a licensing agreement with the incumbent in case of infringement on the incumbent's patent. However, many industries are characterized by precommitment in R&D investment or licensing agreements among firms. Therefore, it is important to investigate if, and how the effects of an increase in scope depend on these assumptions. In this section, each of the two assumptions will be relaxed in turn.

8.1 Stackelberg competition

Suppose that the incumbent can commit to an investment in R&D. For example, he builds a new research lab or employs researchers. The incumbent then acts as a Stackelberg leader. The entrant observes the incumbent's investment, and then decides which technology to invest in, and how much to invest. In the equilibrium of type C, the firms' optimal investments are dependent on each other. If the incumbent is a Stackelberg leader, he can affect the entrant's optimal investment level. In addition, the incumbent can affect the entrant's technology choice. If the incumbent's investment is sufficiently large, the entrant will get a higher expected payoff from choosing technology A than from choosing C. Hence, by sufficient overinvestment, the incumbent can keep the entrant out of technology C. When the equilibrium is of type A, the firms' investments are independent of each other. Hence, unlike in equilibrium of type C, the incumbent is not able to affect the entrant's optimal investment level in this equilibrium by moving first. The entrant optimally invests $p_E^A(\gamma_C)$ under both Stackelberg competition and simultaneous moves.

8.1.1 Investments in equilibrium of type C

When the incumbent invests first, he takes into account the entrant's optimal response to his investment. Since the entrant's investment is no longer taken as given, there is an additional effect of the incumbent's investment on his expected payoff, through the entrant's investment. As shown in (3), the entrant's reaction function is decreasing in the investment by the incumbent. This implies that by investing more, the incumbent not only increases his probability of winning, but also indirectly decreases the entrant's probability of winning as the entrant is induced to invest less.

Let the optimal investments by the two firms in equilibrium of type C be $p_{I,S}$ and $p_{E,S}$ where subscript S denotes Stackelberg competition. In order to find the optimal investment by the incumbent, I insert (3) into (1). Taking the first order condition yields

$$p_{I,S}(\alpha, \gamma_C) = \frac{\gamma_C \left(1 - \alpha - \frac{1}{2}\gamma_C + \frac{3}{2}\alpha\gamma_C\right)}{\left(1 - \frac{1}{2}\gamma_C^2 + \alpha\gamma_C^2\right)} \tag{11}$$

The optimal investment by the entrant is (as given by (11) and (3))

$$p_{E,S}(\alpha, \gamma_C) = \frac{\gamma_C \left(2\alpha\gamma_C - 2\gamma_C - \gamma_C^2 + \alpha\gamma_C^2 + 4\right)}{2\left(2\alpha\gamma_C^2 - \gamma_C^2 + 2\right)}$$

Since the entrant's reaction function is decreasing in the incumbent's investment, the following holds.

Proposition 4 *For all* $\alpha \in [0,1]$ *and all* $\gamma_C \in \left[\frac{1}{2}, 1\right]$, $p_{I,S}(\alpha, \gamma_C) > p_I^C(\alpha, \gamma_C)$

Proof. See Appendix. ■

In an equilibrium of type C, if the incumbent is a Stackelberg leader, he always invests more than when the two firms move simultaneously.

8.1.2 The entrant's choice of technology

The level of investment by the incumbent which induces the entrant to choose technology A is denoted $\bar{p}_I$ and can be expressed as

$$\bar{p}_I(\gamma_C) = \frac{2\,(2\gamma_C - 1)}{\gamma_C} \tag{12}$$

The investment level $\bar{p}_I(\gamma_C)$ is increasing in γ_C. The higher is the relative advantage of technology C, the larger is the investment required to keep the entrant out of it. Note that if $\gamma_C > \frac{2}{3}$ not even the maximal investment by the incumbent, $\bar{p}_I(\gamma_C) = 1$, can prevent the entrant from choosing technology C under a narrow patent scope, and the equilibrium is always C.

8.1.3 Equilibria under narrow patent scope

Suppose that the patent scope is narrow. In order to establish which equilibrium arises under Stackelberg competition, it is necessary to determine which investment level by the incumbent gives him the highest expected payoff $\Pi_I(i, p_I, p_E)$, given the entrant's optimal response to that investment level, both as regards the latter's technology choice and investment level.

I define a threshold $\bar{\alpha}_S \in (0,1)$, where S denotes Stackelberg competition, such that the incumbent's payoff in the two types of equilibria are equal:

$$\Pi_I(A, p_I(\bar{\alpha}_S, \gamma_C), p_E(\gamma_C)) = \Pi_I(C, p_{I,S}(\bar{\alpha}_S, \gamma_C), p_{E,S}(\bar{\alpha}_S, \gamma_C))$$

Proposition 5 *If* $\alpha < \bar{\alpha}_S$ *the equilibrium is of type* A*, and if* $\alpha \geq \bar{\alpha}_S$ *the equilibrium is of type* C*.*

Proof. See Appendix ■

If $\alpha < \bar{\alpha}_S$ the incumbent will choose to strategically overinvest, and thereby he induces the entrant to choose to do R&D on technology A. If $\alpha \geq \bar{\alpha}_S$ the incumbent finds it optimal to not to strategically overinvest, and the entrant chooses technology C. The incumbent's payoff functions for different levels of α are shown in Figure 3 a and 3 b.

Figure 3 a

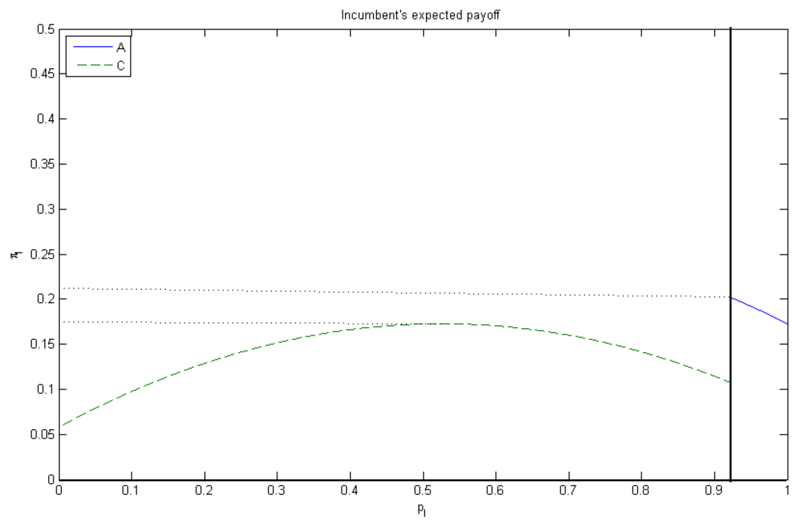
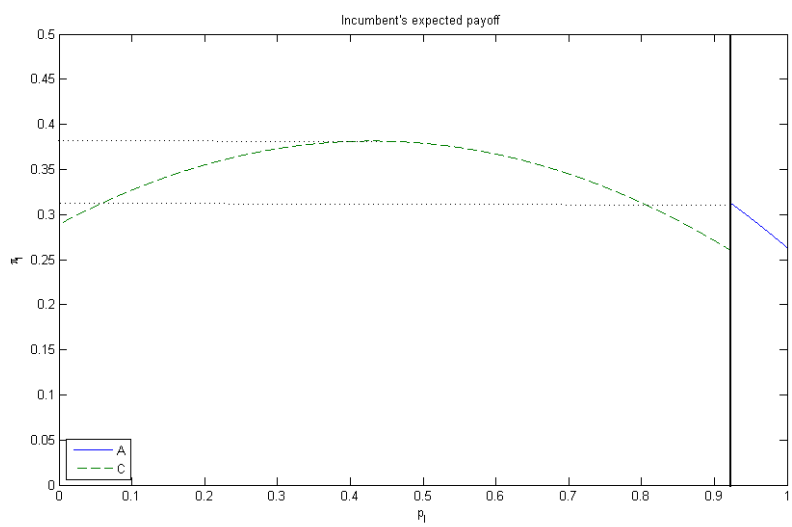


Figure 3 b



Both Figures 3 a and 3 b depict the incumbent's expected payoff Π_I as a function of his investment in equilibrium candidates C and A. The vertical line indicates the threshold level of investment $\bar{p}_I(\gamma_C)$, at which the entrant chooses technology A. Note that the incumbent's investment in equilibrium candidate A is constrained to $\bar{p}_I(\gamma_C)$ whereas in equilibrium candidate C he can invest the optimal level; $p_{I,S}(\alpha, \gamma_C)$. Figure 3a displays the case when $\alpha < \bar{\alpha}_S$. Even though investment is not at its optimal level in A, the expected payoff in A is larger than in C and the incumbent prefers A. Consequently he will strategically overinvest such that the entrant chooses technology A. Figure 3b displays the case when $\alpha > \bar{\alpha}_S$. As α increases, the incumbent's incentives for innovation decrease, and he prefers to invest less. However, it is only in equilibrium of type C that he can reduce his investment, since in equilibrium of type A he must invest at least $\bar{p}_I(\gamma_C)$. Hence, the payoff of choosing A relative to C decreases. For α above the threshold $\bar{\alpha}_S$ the incumbent has a higher expected payoff in equilibrium of type C and will induce the entrant to choose C.

8.1.4 Effects of patent scope

If the patent scope is broad, the Stackelberg competition has no effect on equilibrium investments, since the equilibrium is of type A. The innovation probability is identical to that under a broad scope with simultaneous moves, given by (9).

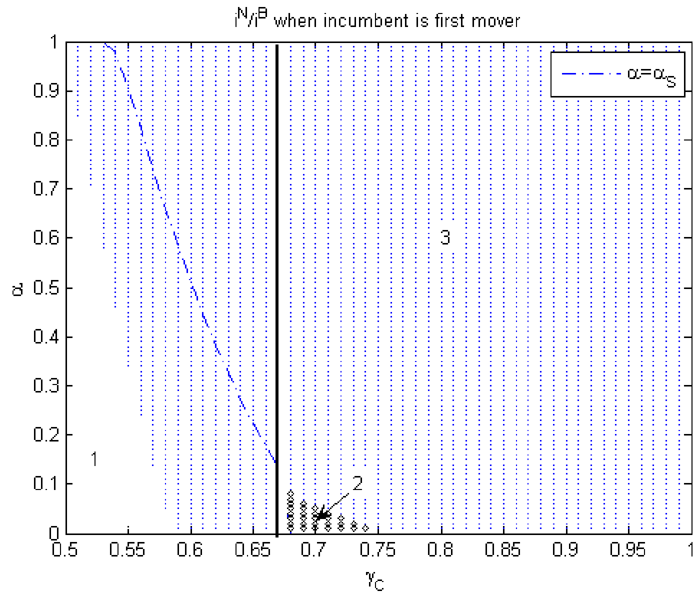
If the patent scope is narrow, the probability of innovation depends on which type of equilibrium firms are in. Let $i^{N,S}$ denote the innovation probability under a narrow scope. If the equilibrium is of type A, there is no duplication of R&D, and hence the only difference between the patent regimes is that under a narrow scope, the incumbent strategically overinvests, and $\bar{p}_I(\gamma_C) > p_I^A(\alpha, \gamma_C)$.[4] Hence, total investment in R&D is higher under a narrow scope, and it follows that $i^{N,S} > i^B$. If the equilibrium is of type C, the innovation probability is

$$i^{N,S} = \gamma_C \left[p_{I,S}(\alpha, \gamma_C) + p_{E,S}(\alpha, \gamma_C) - p_{I,S}(\alpha, \gamma_C) p_{E,S}(\alpha, \gamma_C) \right]$$

In this equilibrium, there is duplication of R&D, as under simultaneous moves. Which patent scope gives the highest innovation probability depends on α and γ_C, as shown in Figure 4.

[4] If $\bar{p}_I(\gamma_C) \leq p_I^A(\alpha, \gamma_C)$, then the entrant would choose technology A even under a narrow patent scope, and patent scope has no effect on innovation probabilities.

Figure 4



Area 1: patent scope is inconsequential.Area 2: $i^{N,S} < i^B$.Area 3: $i^{N,S} > i^B$.

Figure 4 shows that a broad scope gives a higher probability of innovation for a small subset of parameter space, denoted area 2. This holds for values of α close to zero, and values of γ_C close to 0.7. In this area, the incumbent is not able to overinvest and keep the entrant out, since $\gamma_C > \frac{2}{3}$, and the equilibrium is C. However, the incumbent can still affect the entrant's optimal investment level, and the first mover advantage implies that the incumbent invests more, and the entrant less, relative to under simultaneous moves. Now, the negative effects of duplication are sufficiently large that a broad scope gives a higher probability of innovation. To the left of area 2, the equilibrium under a narrow scope is of type A, as the incumbent chooses to overinvest sufficiently to keep the entrant out of technology C. There is no duplication, and a narrow scope gives a higher probability of innovation. To the right of area 2, a higher value of γ_C increases total investments under a narrow scope sufficiently to render it a higher innovation probability than a broad.

If we compare Figures 1 and 4 it is clear that the subset of parameter space for which a broad patent scope gives a higher innovation probability is now substantially smaller. The conclusion is that the effect of patent scope on the innovation probability depends on whether the incumbent can commit to investing or not. If commitment is possible, the potential benefit of a broad scope is substantially smaller.

8.2 Licensing

Until now, any licensing agreement between the two firms has been precluded. It has been assumed that the incumbent always chooses to block the entrant's innovation, if it infringes on his patent. However, if the two firms can write a license agreement, the incumbent may choose to license its technology to the entrant, in return for a license fee. Suppose that the patent on technology C is broad in scope, but the entrant nevertheless chooses to do R&D on technology C. If he innovates and the incumbent does not, the incumbent has two options: he can block the entrant's innovation, and earn his current monopoly profit, αV, or he can license his technology to the entrant, lose current profit but earn a license revenue in form of a fixed fee, F. I assume that the agreement is written ex post, after the entrant has innovated. If the incumbent agrees to license, it implies that the entrant has two strategies available under a broad patent scope. Either he chooses technology A, which gives him V if he innovates, or he chooses technology C which gives him V less the license fee F if he innovates.

The license fee will be determined by bargaining between the licensor and the licensee. The share of surplus from the licensing agreement that goes to each firm depends on its outside option and its relative bargaining power. The incumbent's outside option is to continue selling his patented product, with profit αV. The entrant's outside option is his expected payoff from choosing to do R&D on technology A instead. First, suppose that the incumbent has all the bargaining power. Then he will demand a license fee such that the entrant receives only his outside option, and the entrant always chooses to do R&D on technology A. This implies that if the incumbent has all bargaining power, allowing for license agreements has no effect on equilibrium investments nor innovation probabilities. This maximum fee gives the lower bound on the effects of licensing agreements on investments, which is zero. If, on the other hand, the entrant has all bargaining power, the incumbent will receive a fee equal to his outside option, $F = \alpha V$. The lower is the license fee, the more likely is it that the entrant will choose technology C. Hence, the minimum license fee gives the upper bound on the effects of licensing agreements. If the bargaining powers lie in between these two extremes, the effect of licensing on investments and innovation falls between zero and the upper bound. In order to find the upper bound, I determine the effect of licensing on equilibrium outcomes for the minimum license fee $F = \alpha V$. [5]

8.2.1 Equilibrium of type C

The expected payoff to the incumbent when both firms choose technology C and the entrant obtains a license in case he innovates is

$$\begin{aligned}\Pi_{I,L}(C, p_I, p_E) &= \alpha V + \gamma_C p_I (1 - p_E)(V - \alpha V) + \gamma_C p_E (1 - p_I)(0) \\ &\quad + \gamma_C p_E p_I \left(\frac{1}{2}(V - \alpha V) + \frac{1}{2}(0) \right) - \frac{(p_I)^2}{2}\end{aligned}$$

[5] As in the baseline model, I assume that if indifferent, the entrant chooses technology A.

where the subscript L denotes licensing. The difference between this expected payoff and (1) is that in case the entrant wins, the incumbent licenses the technology, gets license fee αV and loses current profit αV. The net gain is zero. In case both innovate and the entrant gets the patent, the net gain is again zero.

The expected payoff to the entrant when both firms choose technology C and the entrant obtains a license in case he innovates is

$$\Pi_{E,L}(C, p_I, p_E) = \gamma_C p_E (1 - p_I)(V - \alpha V) + \gamma_C p_E p_I \frac{1}{2}(V - \alpha V) - \frac{(p_E)^2}{2}$$

The difference between this expected payoff and (2) is that the entrant, in case he wins must pay a license fee, and the net gain is $V - \alpha V$. Solving for the Nash equilibrium yields:

$$p_{I,L}(\alpha, \gamma_C) = p_{E,L}(\alpha, \gamma_C) = \frac{2\gamma_C(1-\alpha)}{2 + \gamma_C(1-\alpha)}$$

The optimal investments for the entrant and the incumbent are identical. The reason is that through the license fee, the entrant indirectly takes into account the incumbent's profit loss. In addition, the incumbent's expected payoff from not innovating when the entrant does is zero since the license revenue compensates him for the loss of current profit. Comparing these investments to their counterparts in equilibrium C in the baseline model, (4) and (5), I find that for all $\alpha > 0$,

$$p_{I,L}(\alpha, \gamma_C) < p_I^C(\alpha, \gamma_C) \quad (13)$$

$$p_{E,L}(\alpha, \gamma_C) < p_E^C(\alpha, \gamma_C) \quad (14)$$

The entrant invests less under licensing because the net reward to innovation is lower, and the incumbent invests less because he has less to lose from not innovating.

8.2.2 The entrant's choice of technology

Under licensing, the entrant chooses between doing R&D on technology C which has a higher probability of success, but where the payoff is reduced to $V - \alpha V$ or technology A which has a lower probability of success but yields a payoff of V. It is possible to derive a condition for when the entrant chooses technology C over A.

Proposition 6 *Let the licensing fee be αV. The entrant chooses technology C even under a broad patent scope if*

$$\alpha < \frac{3\gamma_C - 2 + \gamma_C^2}{\gamma_C + \gamma_C^2} = \bar{\alpha}_L \quad (15)$$

Proof. See Appendix ■

The entrant chooses technology C for sufficiently low α, that is when the license fee is sufficiently low. The threshold $\bar{\alpha}_L$, where L denotes licensing, is increasing in γ_C. A higher advantage for technology C relative to A increases the payoff to the entrant from choosing C.

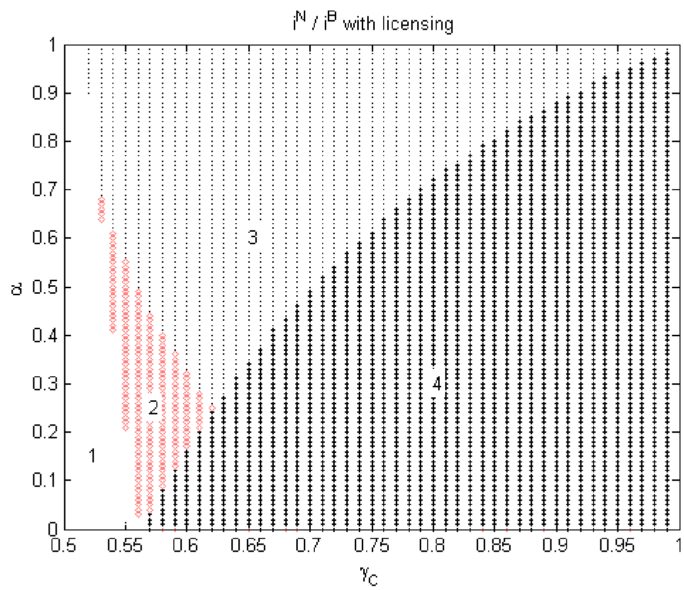
8.2.3 Effects of patent scope

When licensing is allowed, a broad patent scope does not necessarily reduce duplication. When (15) holds, the entrant chooses technology C even though a new innovation would infringe on the patent. Hence, the two firms invest in the same technology even under a broad patent scope. The probability of innovation is:

$$i^{B,L} = \gamma_C \left[p_{I,L}(\alpha, \gamma_C) + p_{E,L}(\alpha, \gamma_C) - p_{I,L}(\alpha, \gamma_C)\, p_{E,L}(\alpha, \gamma_C) \right]$$

where superscript L denotes licensing. Under a narrow scope, no license is required and the innovation probability is identical to that under no licensing; as given by (8). The subset of parameter space where the entrant chooses to do R&D on technology C and obtain a license is shown in Figure 5.

Figure 5



Area 1: patent scope is inconsequential.Area 2: $i^N < i^B$.Area 3: $i^N > i^B$.Area 4: (15) holds and $i^N > i^{B,L}$.

In Figure 5, the areas 1, 2 and 3 are the subsets of parameter space where under a broad scope, the entrant chooses technology A even when he has the option to get a license. The innovation probabilities are unaffected by the licensing option. In area 4, the entrant finds it profitable to use the licensing option and chooses C. For this subset of parameter space, the innovation probability is higher under a narrow scope. The reason is that there is now duplication of R&D under both narrow and broad scope and in addition, the incumbent and entrant both invest less under a broad scope, as shown by (13) and (14). [6]

Comparing Figures 1 and 5, one can conclude that the area of the parameter space where a broad scope gives a higher probability of innovation is smaller with licensing than without. If licensing is possible and the entrant has some bargaining power, the benefit of a broad patent scope is smaller.

9 Concluding comments

The model developed in this paper is motivated by the perceived increase in patent scope in the US in the last two decades. The model predicts the level of investment in R&D and the innovation probability resulting from a narrow and broad patent scope respectively. It suggests a new explanation for the empirical fact that incumbent firms have high innovation rates relative to entrant firms. An incumbent firm can be more likely to innovate even in absence of any technological or cost advantage, if the firm has an advantage originating from policy, namely a broad scope on the patent he owns.

The main finding is that if the incumbent has a high stand-alone incentive to innovate and the patented technology has a small advantage relative to alternative technologies, a broad patent scope gives a higher probability of innovation than a narrow. Consequently, the negative effects of duplication of R&D investments are under some conditions large enough to warrant a broad patent scope. Conversely, when the incumbent's stand-alone incentive is low or the patented technology has a large advantage, a narrow patent scope gives a higher probability of innovation. When the incumbent can commit to an investment level, or when license agreements can be made, the first result is partly reversed; in instances where previously the highest innovation probability was given by a broad scope, it is now obtained under a narrow scope. Consequently, the benefit of a broad patent scope largely relies on the assumptions that the firms act simultaneously and that there is no possibility for licensing agreements.

This paper shows that the effects of an increase in patent scope depends on innovation and industry characteristics. It is possible that an increase in patent scope increases the probability of innovation in a given industry. However, it requires that specific conditions on the form of competition, the technological

[6] Figure 5 depicts the maximal effect of licensing agreements, which is when the entrant has all the bargaining power. If the incumbent has some bargaining power, that increases the license fee, which shifts area 4 to the right. If the incumbent has all the bargaining power, area 4 disappears completely and licensing has no effect on innovation probabilities.

alternatives, the expected profits and the opportunities for license agreements are met. If not, the result is a reduction in the probability of innovation. A uniform increase in patent scope, such as awarding patent holders larger powers in infringement lawsuits, cannot be an optimal policy.

The conclusion raises a new question: is the optimal policy implementable? In order to set the optimal scope ex ante, the Patent Office must make predictions of for example the technology's advantage relative to alternatives and the patent holder's incentive for further improvement of the innovation he seeks to patent. This might seem as an inherently difficult task for the patent examiner. However, the patent scope is also determined ex post, if the patent holder sues another party for infringement. At this point in time, the industry characteristics are observed rather than predicted. The court, deciding whether a product infringes on the patent or not, can at least in principle obtain information on alternative technologies and the incumbent's stand-alone incentive for innovation. If the court finds that a narrow scope would have generated a higher rate of innovation, it should decide that the product was not infringing on the patent. If entrant firms anticipate such a decision by the court, they will make the desirable technology choice.

A direction for future research is to increase the realism of the model by extending it to a dynamic framework, where the effects of expectations and the dynamics of technology development can be analyzed.

Appendix

Proof of proposition 1

To compute the expected payoff to the entrant in equilibrium of type C I insert the equilibrium investments $p_I^C(\alpha,\gamma_C) = \frac{2\gamma_C(2(1-\alpha)+2\alpha\gamma_C-\gamma_C)}{(4+2\alpha\gamma_C^2-\gamma_C^2)}$, $p_E^C(\alpha,\gamma_C) = \frac{2\gamma_C(2+\alpha\gamma_C-\gamma_C)}{(4+2\alpha\gamma_C^2-\gamma_C^2)}$ into

$$\Pi_E(C,p_I,p_E) = \gamma_C p_E(1-p_I)V + \gamma_C p_E p_I \frac{1}{2}V - b\frac{(p_E)^2}{2}$$

Given $V=1$,the expression can be simplified to

$$\Pi_E\left(C,p_I^C(\alpha,\gamma_C),p_E^C(\alpha,\gamma_C)\right) = \frac{2\gamma_C^2(2+\alpha\gamma_C-\gamma_C)^2}{(4+2\alpha\gamma_C^2-\gamma_C^2)^2}$$

To compute the expected payoff to the entrant in equilibrium of type A, I insert $p_E^A(\gamma_C) = (1-\gamma_C)$ into

$$\Pi_E(A,p_I,p_E) = (1-\gamma_C)p_E V - \frac{(p_E)^2}{2}$$

Given $V=1$,the expression can be simplified to

$$\Pi_E\left(A,p_I,p_E^A(\gamma_C)\right) = \frac{1}{2}(1-\gamma_C)^2$$

The entrant chooses technology C if

$$\begin{aligned} \Pi_E\left(C,p_I^C(\alpha,\gamma_C),p_E^C(\alpha,\gamma_C)\right) &> \Pi_E\left(A,p_I,p_E^A(\gamma_C)\right) \\ \frac{2\gamma_C^2(2+\alpha\gamma_C-\gamma_C)^2}{(4+2\alpha\gamma_C^2-\gamma_C^2)^2} &> \frac{1}{2}(1-\gamma_C)^2 \end{aligned}$$

which can be simplified to

$$\alpha > \frac{4-8\gamma_C+\gamma_C^2+\gamma_C^3}{2\gamma_C^3}$$

Proof of proposition 2

$$\frac{i^N}{i^B} = \frac{2\gamma_C^2\left(20\alpha\gamma_C-16\gamma_C-8\alpha+4\gamma_C^2-6\alpha\gamma_C^2-\alpha\gamma_C^3+2\alpha^2\gamma_C^3+16\right)}{\left(\gamma_C^2(1-\alpha)+(1-\gamma_C)^2\right)(4+2\alpha\gamma_C^2-\gamma_C^2)^2}$$

The derivative of $\frac{i^N}{i^B}$ with respect to α is

$$\frac{\partial\left(\frac{i^N}{i^B}\right)}{\partial\alpha} = 2\gamma_C^2\frac{\begin{pmatrix} 144\gamma_C-240\gamma_C^2+296\gamma_C^3-250\gamma_C^4+117\gamma_C^5-26\gamma_C^6+2\gamma_C^7 \\ +16\alpha\gamma_C^2-56\alpha\gamma_C^3+188\alpha\gamma_C^4-170\alpha\gamma_C^5+52\alpha\gamma_C^6-4\alpha\gamma_C^7 \\ -32\alpha^2\gamma_C^4+72\alpha^2\gamma_C^5-24\alpha^2\gamma_C^6-2\alpha^2\gamma_C^7+4\alpha^3\gamma_C^7-32 \end{pmatrix}}{(2\alpha\gamma_C^2-\gamma_C^2+4)^3(2\gamma_C-2\gamma_C^2+\alpha\gamma_C^2-1)^2}$$

The denominator of the expression above is positive, since $4 > \gamma_C^2$. Let

$$\begin{aligned} F(\gamma_C,\alpha) &= 144\gamma_C - 240\gamma_C^2 + 296\gamma_C^3 - 250\gamma_C^4 + 117\gamma_C^5 - 26\gamma_C^6 + 2\gamma_C^7 + 16\alpha\gamma_C^2 \\ &\quad -56\alpha\gamma_C^3 + 188\alpha\gamma_C^4 - 170\alpha\gamma_C^5 + 52\alpha\gamma_C^6 - 4\alpha\gamma_C^7 - 32\alpha^2\gamma_C^4 + 72\alpha^2\gamma_C^5 \\ &\quad -24\alpha^2\gamma_C^6 - 2\alpha^2\gamma_C^7 + 4\alpha^3\gamma_C^7 - 32 \end{aligned}$$

$\frac{\partial\left(\frac{i^N}{i^B}\right)}{\partial\alpha}$ is positive if $F(\gamma_C,\alpha) > 0$. Applying constrained optimization, the problem can be formulated as

$$\min_{\gamma_C,\alpha} F(\gamma_C,\alpha) \quad \textit{subject to} \quad 0 \leqslant \alpha \leqslant 1 \quad 0.5 \leqslant \gamma_C \leqslant 1$$

where $F(\gamma_C,\alpha)$ is continuously differentiable. The global minimum of $F(\gamma_C,\alpha)$ is $F(0.5,0) = 4.6$. Hence, for $\gamma_C \in [0.5,1]$ and $\alpha \in [0,1]$ we have that $\frac{\partial\left(\frac{i^N}{i^B}\right)}{\partial\alpha} > 0$.

Proof of proposition 3

The derivative of $\frac{i^N}{i^B}$ with respect to γ_C is

$$\frac{\partial\left(\frac{i^N}{i^B}\right)}{\partial\gamma_C} = 2\gamma_C \frac{\left(\begin{array}{c} 304\alpha\gamma_C - 320\gamma_C - 64\alpha + 352\gamma_C^2 - 336\gamma_C^3 + 256\gamma_C^4 - 104\gamma_C^5 + 16\gamma_C^6 \\ -496\alpha\gamma_C^2 + 640\alpha\gamma_C^3 - 688\alpha\gamma_C^4 + 365\alpha\gamma_C^5 - 64\alpha\gamma_C^6 - 2\alpha\gamma_C^7 + 32\alpha^2\gamma_C^2 \\ -176\alpha^2\gamma_C^3 + 432\alpha^2\gamma_C^4 - 364\alpha^2\gamma_C^5 - 64\alpha^3\gamma_C^4 + 76\alpha^2\gamma_C^6 + 100\alpha^3\gamma_C^5 \\ +9\alpha^2\gamma_C^7 - 24\alpha^3\gamma_C^6 - 12\alpha^3\gamma_C^7 + 4\alpha^4\gamma_C^7 + 128 \end{array}\right)}{\left(2\alpha\gamma_C - \gamma_C^2 + 4\right)^3\left(2\gamma_C - 2\gamma_C^2 + \alpha\gamma_C^2 - 1\right)^2}$$

The denominator of this expression is positive since $4 > \gamma_C^2$. Let

$$\begin{aligned} G(\gamma_C,\alpha) &= 304\alpha\gamma_C - 320\gamma_C - 64\alpha + 352\gamma_C^2 - 336\gamma_C^3 + 256\gamma_C^4 - 104\gamma_C^5 + 16\gamma_C^6 \\ &\quad -496\alpha\gamma_C^2 + 640\alpha\gamma_C^3 - 688\alpha\gamma_C^4 + 365\alpha\gamma_C^5 - 64\alpha\gamma_C^6 - 2\alpha\gamma_C^7 \\ &\quad +32\alpha^2\gamma_C^2 - 176\alpha^2\gamma_C^3 + 432\alpha^2\gamma_C^4 - 364\alpha^2\gamma_C^5 - 64\alpha^3\gamma_C^4 + 76\alpha^2\gamma_C^6 \\ &\quad +100\alpha^3\gamma_C^5 + 9\alpha^2\gamma_C^7 - 24\alpha^3\gamma_C^6 - 12\alpha^3\gamma_C^7 + 4\alpha^4\gamma_C^7 + 128 \end{aligned}$$

At $\gamma_C = 0$ the expression reduces to

$$G(\gamma_C,\alpha) = 128 - 64\alpha$$

which is positive for all $\alpha \in [0,1]$. At $\gamma_C = 1$ the expression reduces to

$$G(\gamma_C,\alpha) = 9\alpha^2 - 5\alpha + 4\alpha^4 - 8$$

which is negative for all $\alpha \in [0,1)$. $G(\gamma_C,\alpha)$ is continuous, and the intermediate value theorem can be applied. Hence, for $\alpha \in [0,1)$ there exists at least one maximum of the function $\frac{i^N}{i^B}$ in the interval $\gamma_C \in (0,1)$.

Proof of proposition 4

$$\begin{aligned} p_{I,S}(\alpha,\gamma_C) &= \frac{\gamma_C\left(1 - \alpha - \frac{1}{2}\gamma_C + \frac{3}{2}\alpha\gamma_C\right)}{\left(1 - \frac{1}{2}\gamma_C^2 + \alpha\gamma_C^2\right)} \\ p_I^C(\alpha,\gamma_C) &= \frac{2\gamma_C\left(2(1-\alpha) + 2\alpha\gamma_C - \gamma_C\right)}{\left(4 + 2\alpha\gamma_C^2 - \gamma_C^2\right)} \end{aligned}$$

$$\begin{aligned} p_{I,S}(\alpha,\gamma_C) &> p_I^C(\alpha,\gamma_C) \Leftrightarrow \\ 0 &> \frac{1}{2}\gamma_C\left(6\alpha\gamma_C - 2\gamma_C - 4\alpha + \gamma_C^2 - 3\alpha\gamma_C^2 - 4\alpha^2\gamma_C + 2\alpha^2\gamma_C^2\right) \end{aligned}$$

Let

$$J(\gamma_C,\alpha) = 6\alpha\gamma_C - 2\gamma_C - 4\alpha + \gamma_C^2 - 3\alpha\gamma_C^2 - 4\alpha^2\gamma_C + 2\alpha^2\gamma_C^2$$

Applying constrained optimization, the problem can be formulated as

$$\max_{\gamma_C,\alpha} J(\gamma_C,\alpha) \quad \textit{subject to} \quad 0 \leqslant \alpha \leqslant 1 \quad \tfrac{1}{2} \leqslant \gamma_C \leqslant 1$$

where $J(\gamma_C,\alpha)$ is continuously differentiable. The optimization yields a global maximum of $J(\gamma_C,\alpha)$ at $J(0.5,0) = -0.75 < 0$. It implies that $p_{I,S}(\alpha,\gamma_C) > p_I^C(\alpha,\gamma_C)$.

Proof of proposition 5

In equilibrium candidate of type C, the incumbent invests

$$p_{I,S}^C(\alpha,\gamma_C) = \min(p_{I,S}(\alpha,\gamma_C), \bar{p}_I(\gamma_C))$$

If

$$\alpha > \frac{3\gamma_C^3 - 8\gamma_C + 4}{\gamma_C^2\,(5\gamma_C - 2)} = \hat{\alpha}_1$$

then $p_{I,S}^C(\alpha,\gamma_C) = p_{I,S}(\alpha,\gamma_C)$.

In equilibrium candidate of type A the incumbent invests

$$p_{I,S}^A(\alpha,\gamma_C) = \max(p_I^A(\alpha,\gamma_C), \bar{p}_I(\gamma_C))$$

If

$$\alpha > \frac{\gamma_C^2 - 4\gamma_C + 2}{\gamma_C^2} = \hat{\alpha}_2$$

then $p_{I,S}^A(\alpha,\gamma_C) = \bar{p}_I(\gamma_C)$

There are 4 cases to consider:

Case	$p_{I,S}^C(\alpha,\gamma_C)$	$p_{I,S}^A(\alpha,\gamma_C)$
1	$\alpha < \hat{\alpha}_1$	$\alpha < \hat{\alpha}_2$
2	$\alpha > \hat{\alpha}_1$	$\alpha < \hat{\alpha}_2$
3	$\alpha < \hat{\alpha}_1$	$\alpha > \hat{\alpha}_2$
4	$\alpha > \hat{\alpha}_1$	$\alpha > \hat{\alpha}_2$

I start with case 3, and then proceed to cases 1,2 and 4.

Case 3. Compare $\Pi_I(C, \bar{p}_I(\gamma_C), p_E)$ and $\Pi_I\left(A, \bar{p}_I(\gamma_C), p_E^A(\gamma_C)\right)$. In equilibrium of type $C: p_E = \gamma_C\left(1 - \frac{\bar{p}_I(\gamma_C)}{2}\right)$

$$\begin{aligned}
\Pi_I\left(A, \bar{p}_I(\gamma_C), p_E^A(\gamma_C)\right) &= \frac{\left(-10\gamma_C^2 + 8\gamma_C + 4\gamma_C^3 + 2\alpha\gamma_C^2 - 2\alpha\gamma_C^3 - \alpha\gamma_C^4 - 2\right)}{\gamma_C^2} \\
\Pi_I(C, \bar{p}_I(\gamma_C), p_E) &= \frac{\left(-9\gamma_C^2 + 8\gamma_C + \gamma_C^3 + 2\gamma_C^4 + \alpha\gamma_C^2 + \alpha\gamma_C^3 - 3\alpha\gamma_C^4 - 2\right)}{\gamma_C^2}
\end{aligned}$$

$$\begin{aligned}
\Pi_I\left(A, \bar{p}_I(\gamma_C), p_E^A(\gamma_C)\right) &> \Pi_I(C, \bar{p}_I(\gamma_C), p_E) \Leftrightarrow \\
& \frac{\left(-10\gamma_C^2 + 8\gamma_C + 4\gamma_C^3 + 2\alpha\gamma_C^2 - 2\alpha\gamma_C^3 - \alpha\gamma_C^4 - 2\right)}{\gamma_C^2} \\
&> \frac{\left(-9\gamma_C^2 + 8\gamma_C + \gamma_C^3 + 2\gamma_C^4 + \alpha\gamma_C^2 + \alpha\gamma_C^3 - 3\alpha\gamma_C^4 - 2\right)}{\gamma_C^2}
\end{aligned}$$

which can be simplified to

$$\gamma_C^2\left(2\gamma_C - 1\right)\left(1 - \gamma_C\right)\left(1 - \alpha\right) > 0$$

For $\gamma_C > \frac{1}{2}$ and $\alpha < 1 : \gamma_C^2\left(2\gamma_C - 1\right)\left(1 - \gamma_C\right)\left(1 - \alpha\right) > 0$.The incumbent prefers A.

Case 1. Compare $\Pi_I(C, \bar{p}_I(\gamma_C), p_E)$ and $\Pi_I\left(A, p_I^A(\alpha, \gamma_C), p_E^A(\gamma_C)\right)$

From case 3 it is clear that $\Pi_I\left(A, \bar{p}_I(\gamma_C), p_E^A(\gamma_C)\right) > \Pi_I(C, \bar{p}_I(\gamma_C), p_E)$. In addition, $\Pi_I\left(A, p_I^A(\alpha, \gamma_C), p_E^A(\gamma_C)\right) > \Pi_I(A, \bar{p}_I(\gamma_C), p_E^A(\gamma_C))$ since $p_I^A(\alpha, \gamma_C) = \arg\max_{p_I} \Pi_I(A, p_I, p_E)$. Hence, $\Pi_I\left(A, p_I^A(\alpha, \gamma_C), p_E^A(\gamma_C)\right) > \Pi_I(C, \bar{p}_I(\gamma_C), p_E)$ and the incumbent prefers A.

Case 2. Compare $\Pi_I\left(C, p_{I,S}(\alpha, \gamma_C), p_{E,S}(\alpha, \gamma_C)\right)$ and $\Pi_I\left(A, p_I^A(\alpha, \gamma_C), p_E^A(\gamma_C)\right)$.

$$\begin{aligned}
\Pi_I\left(C, p_{I,S}(\alpha, \gamma_C), p_{E,S}(\alpha, \gamma_C)\right) &= \frac{\begin{pmatrix} 8\alpha + 4\gamma_C^2 - 4\gamma_C^3 + \gamma_C^4 - 20\alpha\gamma_C^2 + 16\alpha\gamma_C^3 - 2\alpha\gamma_C^4 \\ +12\alpha^2\gamma_C^2 - 12\alpha^2\gamma_C^3 + \alpha^2\gamma_C^4 \end{pmatrix}}{4\left(2\alpha\gamma_C^2 - \gamma_C^2 + 2\right)} \\
\Pi_I\left(A, p_I^A(\alpha, \gamma_C), p_E^A(\gamma_C)\right) &= \frac{1}{2}\gamma_C\left(4\alpha + \gamma_C - 4\alpha\gamma_C + \alpha^2\gamma_C\right)
\end{aligned}$$

$$\Pi_I\left(A, p_I^A(\alpha, \gamma_C), p_E^A(\gamma_C)\right) > \Pi_I\left(C, p_{I,S}(\alpha, \gamma_C), p_{E,S}(\alpha, \gamma_C)\right) \Leftrightarrow$$

$$\begin{aligned}
0 > & \\
& \frac{\left(8\alpha + 4\gamma_C^2 - 4\gamma_C^3 + \gamma_C^4 - 20\alpha\gamma_C^2 + 16\alpha\gamma_C^3 - 2\alpha\gamma_C^4 + 12\alpha^2\gamma_C^2 - 12\alpha^2\gamma_C^3 + \alpha^2\gamma_C^4\right)}{4\left(2\alpha\gamma_C^2 - \gamma_C^2 + 2\right)} \\
& -\frac{1}{2}\gamma_C\left(4\alpha + \gamma_C - 4\alpha\gamma_C + \alpha^2\gamma_C\right)
\end{aligned}$$

Let

$$K(\gamma_C,\alpha) = \frac{\left(\begin{array}{c} 8\alpha + 4\gamma_C^2 - 4\gamma_C^3 + \gamma_C^4 - 20\alpha\gamma_C^2 + 16\alpha\gamma_C^3 - 2\alpha\gamma_C^4 + 12\alpha^2\gamma_C^2 \\ -12\alpha^2\gamma_C^3 + \alpha^2\gamma_C^4 \end{array}\right)}{4\left(2\alpha\gamma_C^2 - \gamma_C^2 + 2\right)} - \frac{1}{2}\gamma_C\left(4\alpha + \gamma_C - 4\alpha\gamma_C + \alpha^2\gamma_C\right)$$

I want to find the maximum value of $K(\gamma_C, \alpha)$. Applying constrained optimization, the problem can be formulated as

$$\max_{\gamma_C,\alpha} K(\gamma_C,\alpha) \quad \textit{subject to} \quad 0 \leqslant \alpha \leqslant 0.28 \quad 0.55 \leqslant \gamma_C \leqslant 0.59$$

where $K(\gamma_C, \alpha)$ is continuously differentiable. The optimization yields a global maximum of $K(\gamma_C, \alpha)$ at $K(0.55, 0.28) = -0.03 < 0$. It implies that $\Pi_I\left(A, p_I^A(\alpha,\gamma_C), p_E^A(\gamma_C)\right) > \Pi_I\left(C, p_{I,S}(\alpha,\gamma_C), p_{E,S}(\alpha,\gamma_C)\right)$ and the incumbent prefers A.

Case 4. Compare $\Pi_I\left(C, p_{I,S}(\alpha,\gamma_C), p_{E,S}(\alpha,\gamma_C)\right)$ and $\Pi_I\left(A, \bar{p}_I(\gamma_C), p_E^A(\gamma_C)\right)$

$$\Pi_I\left(C, p_{I,S}(\alpha,\gamma_C), p_{E,S}(\alpha,\gamma_C)\right) > \Pi_I\left(A, \bar{p}_I(\gamma_C), p_E^A(\gamma_C)\right) \Leftrightarrow$$

$$\frac{\left(8\alpha + 4\gamma_C^2 - 4\gamma_C^3 + \gamma_C^4 - 20\alpha\gamma_C^2 + 16\alpha\gamma_C^3 - 2\alpha\gamma_C^4 + 12\alpha^2\gamma_C^2 - 12\alpha^2\gamma_C^3 + \alpha^2\gamma_C^4\right)}{4\left(2\alpha\gamma_C^2 - \gamma_C^2 + 2\right)} > \frac{\left(-10\gamma_C^2 + 8\gamma_C + 4\gamma_C^3 + 2\alpha\gamma_C^2 - 2\alpha\gamma_C^3 - \alpha\gamma_C^4 - 2\right)}{\gamma_C^2}$$

Let

$$\bar{\alpha}_S = \frac{\left(\begin{array}{c} -4\gamma_C^2 + 24\gamma_C^3 - 38\gamma_C^4 + 12\gamma_C^5 + 3\gamma_C^6 \\ +2\sqrt{20\gamma_C^4 - 128\gamma_C^5 + 320\gamma_C^6 - 408\gamma_C^7 + 301\gamma_C^8 - 144\gamma_C^9 + 49\gamma_C^{10} - 10\gamma_C^{11}} \end{array}\right)}{-4\gamma_C^4 + 4\gamma_C^5 + 9\gamma_C^6}$$

For

$$\begin{aligned} \alpha &\geq \bar{\alpha}_S : \Pi_I\left(C, p_{I,S}(\alpha,\gamma_C), p_{E,S}(\alpha,\gamma_C)\right) > \Pi_I\left(A, \bar{p}_I(\gamma_C), p_E^A(\gamma_C)\right) \\ \alpha &< \bar{\alpha}_S : \Pi_I\left(C, p_{I,S}(\alpha,\gamma_C), p_{E,S}(\alpha,\gamma_C)\right) \leq \Pi_I\left(A, \bar{p}_I(\gamma_C), p_E^A(\gamma_C)\right) \end{aligned}$$

I assume that if indifferent, the incumbent chooses C.

Show that $\alpha_1 < \bar{\alpha}_S$ for $\gamma_C \in [0.5332, 0.667]$

$$\alpha_1 < \bar{\alpha}_S \Leftrightarrow$$

$$\frac{3\gamma_C^3 - 8\gamma_C + 4}{\gamma_C^2\left(5\gamma_C - 2\right)} < \frac{\left(\begin{array}{c} -4\gamma_C^2 + 24\gamma_C^3 - 38\gamma_C^4 + 12\gamma_C^5 + 3\gamma_C^6 \\ +2\sqrt{\begin{array}{c} 20\gamma_C^4 - 128\gamma_C^5 + 320\gamma_C^6 - 408\gamma_C^7 + 301\gamma_C^8 \\ -144\gamma_C^9 + 49\gamma_C^{10} - 10\gamma_C^{11} \end{array}} \end{array}\right)}{-4\gamma_C^4 + 4\gamma_C^5 + 9\gamma_C^6}$$

the expression can be simplified to

$$0 < 8\gamma_C^4\left(1-\gamma_C\right)\left(2\gamma_C - 1\right)\left(4\gamma_C + 9\gamma_C^2 - 4\right)\left(4\gamma_C - \gamma_C^2 + \gamma_C^3 - 2\right)\left(2\gamma_C^2 - 6\gamma_C + \gamma_C^3 + 4\right)$$

which holds for $\gamma_C \in [0.5332, 0.6667]$.

Show that $\alpha_2 < \bar{\alpha}_S$ for $\gamma_C \in [0.5332, 0.6667]$:

$$\alpha_2 > \bar{\alpha}_S \Leftrightarrow$$

$$\frac{\gamma_C^2 - 4\gamma_C + 2}{\gamma_C^2} < \frac{\left(\begin{array}{c} -4\gamma_C^2 + 24\gamma_C^3 - 38\gamma_C^4 + 12\gamma_C^5 + 3\gamma_C^6 \\ +2\sqrt{\begin{array}{c} 20\gamma_C^4 - 128\gamma_C^5 + 320\gamma_C^6 - 408\gamma_C^7 + 301\gamma_C^8 \\ -144\gamma_C^9 + 49\gamma_C^{10} - 10\gamma_C^{11} \end{array}} \end{array}\right)}{-4\gamma_C^4 + 4\gamma_C^5 + 9\gamma_C^6}$$

The expression can be simplified to

$$392\gamma_C^2 - 128\gamma_C - 496\gamma_C^3 - 11\gamma_C^4 + 648\gamma_C^5 - 543\gamma_C^6 + 122\gamma_C^7 - 9\gamma_C^8 + 16 > 0 \quad (16)$$

Let

$$f_{\alpha_2}(\gamma_C) = 392\gamma_C^2 - 128\gamma_C - 496\gamma_C^3 - 11\gamma_C^4 + 648\gamma_C^5 - 543\gamma_C^6 + 122\gamma_C^7 - 9\gamma_C^8 + 16$$

$\frac{\partial f_{\alpha_2}(\gamma_C)}{\partial \gamma_C} > 0 \;\; for \;\; \gamma_C \in [0.5332, 0.6667]$ and $f_{\alpha_2}(0.5332) = 4.39 \times 10^{-2} > 0$. It follows that for $\alpha < \bar{\alpha}_S$, the equilibrium is of type A and for $\alpha \geq \bar{\alpha}_S$ the equilibrium is of type C.

Proof of proposition 6

The expected payoff to the entrant in equilibrium A is

$$\Pi_E\left(A, p_I, p_E^A(\gamma_C)\right) = \frac{1}{2}\left(1-\gamma_C\right)^2$$

The expected payoff to the entrant from choosing technology C when the patent scope is broad and the incumbent demands a license fee αV is obtained by inserting $p_{E,L} = \frac{2\gamma_C(1-\alpha)}{2+\gamma_C(1-\alpha)}$ and $p_{I,L} = \frac{2\gamma_C(1-\alpha)}{2+\gamma_C(1-\alpha)}$ into

$$\Pi_{E,L}\left(C, p_I, p_E\right) = \gamma_C p_E(1-p_I)(V - \alpha V) + \gamma_C p_E p_I \frac{1}{2}(V - \alpha V) - \frac{\left(p_E\right)^2}{2}$$

which can be simplified to

$$\Pi_{E,L}\left(C, p_{E,L}, p_{I,L}\right) = \frac{2\gamma_C^2 \left(1-\alpha\right)^2}{\left(2+\gamma_C(1-\alpha)\right)^2}$$

The entrant chooses technology C if

$$\begin{aligned} \Pi_{E,L}\left(C, p_{E,L}, p_{I,L}\right) &> \Pi_E\left(A, p_I, p_E^A(\gamma_C)\right) \\ \frac{2\gamma_C^2 \left(1-\alpha\right)^2}{\left(2+\gamma_C(1-\alpha)\right)^2} &> \frac{1}{2}\left(1-\gamma_C\right)^2 \end{aligned}$$

which can be simplified to

$$\alpha < \frac{3\gamma_C - 2 + \gamma_C^2}{\gamma_C + \gamma_C^2}$$

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